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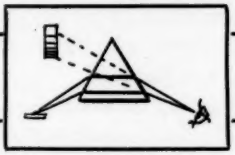

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ERYTHEMA DUE TO ULTRAVIOLET RADIATION

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The recent work on the function of ultraviolet irradiation in the formation of the antirachitic vitamin D has awakened a renewed interest in the health-giving properies of radiations of shorter wave length than the visible spectrum, and in photo-erythema, the reddening of the skin produced by exposure to such radiations.

While the intensity of the radiation in this region is, in general, of first importance and while no strict proportionality between erythema and benefit can be claimed, there are three grounds for an interest in the erythemal effect of the radiation: (1) there is no evidence that radiations of such wave lengths as do not under any circumstances produce erythema have any antirachitic effect; (2) it is desirable to avoid producing a painful degree of erythema, or sunburn, and (3) to avoid undue pigmentation, or tan, which diminishes the effectiveness of subsequent irradiation. There is, again, no strict proportionality between reddening and tan, but the development of pigmentation appears to require the development of some degree of erythema.

Many discussions of the erythemal effectiveness of sources of mixed radiation have been based on a very uncritical use of the data published by Hausser and Vahle[1,2] for the erythemal effectiveness of monochro-

[1] K. W. Hausser & W. Vahle, Strahlentherapie *13*, 41; 1922.
[2] K. W. Hausser & W. Vahle, Wiss. Veroff. aus d. Siemens-Konzern 1927, p. 101.

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matic radiation as a function of the wave length. As commonly used, these may lead to conclusions quite out of harmony with observed fact. (See Table 1). It should especially be pointed out that the *curve* given in the later paper of the degree of erythema produced by equal energy exposures at different wave lengths cannot safely be used for the erythemal effectiveness of mixed radiation at any level of erythema.

The analogy between the photographic action of and the production of erythema by ultraviolet radiation has often been pointed out. This analogy warrants the application to the erythema data of Hausser and Vahle of the methods of analysis[3] which have proved useful in a study of photographic plates, film and paper.

Fig. 1 gives a typical relation between the plate density (after development) and the exposure: a line markedly curved downward. If, instead

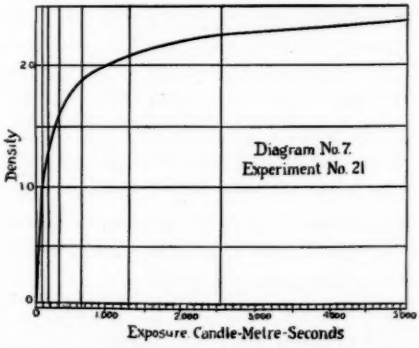

FIG. 1. *Relation between photographic plate density and exposure as given by Hurter & Driffield.*

of exposure (that is, the product of the light intensity and the time of exposure) the logarithm of exposure be used, then as Hurter and Driffield[3] showed, the resulting line is straight in its central portion (called 'the region of correct exposure') with opposite curvatures in the regions of under- and over-exposure. (Fig. 2). The slope of the straight line portion of the 'characteristic curve' they designated by the Greek letter *gamma*.

A variation of *gamma* with wave length means that the relative effectiveness of radiation of different 'colors' depends on the density to which the exposures lead. In the analogous case of the reddening of the human skin by ultraviolet radiation, it will mean that the relative erythema-

[3] F. Hurter & V. C. Driffield, Jour. Soc. Chem. Ind. *9* (5) 455–69; 1890.

producing power of radiation of different wave lengths will depend on the degree of erythema at which the comparisons are made.

The data to which the photographic analogy is to be applied are those given by Hausser[1] in 'Strahlentherapie,' and in somewhat greater detail in a publication of the Siemenskonzern by Hausser and Vahle,[2] for the reddening of untanned human skin by the more important lines or groups of lines in the ultraviolet spectrum of the quartz mercury arc. When the degree of erythema is plotted against relative exposure—as in

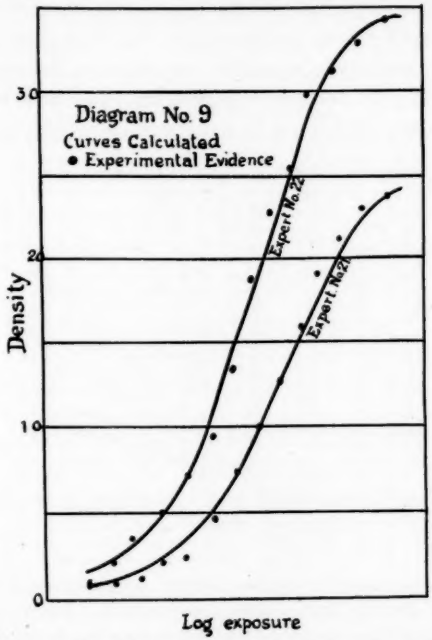

FIG. 2. *Photographic density plotted against the logarithm of exposure (Hurter & Driffield). The straight line portion of the graphs is the 'region of correct exposure.'*

the papers referred to—each wave length gives a curved line similar to that of Fig. 1. Plotting degree of erythema as given by Hausser and Vahle against the logarithm of relative exposure did not give straight lines. This called for a more careful consideration of the 'erythema scale' in which the results are expressed.

In order to make the results as nearly objective as possible, Hausser and Vahle determined the degree of erythema by comparing the color of

[1] K. W Hausser, Strahlentherapie *28*, 25; 1928.

the reddened skin with permanent standards made by coloring 'matte, skin-colored paper' with a dye. (What dye they used is not stated). The paper colored with the maximum concentration of dye corresponded in color to what they called eighth degree erythema. (They do not state what this maximum concentration was). Using half the concentration of dye gave seventh degree, one quarter, 6th, one eighth, 5th, etc. Fractions of degrees were estimated.

Except for the step between zero and first degree erythema, this scale corresponds to the logarithm of the concentration of dye. The concentration of dye, rather than its logarithm, was then plotted as the analog of photographic density against the logarithm of the relative exposure, still the curves did not become straight lines. The most obvious explanation is that the dye used is not an exact match for the hemoglobin of the blood.

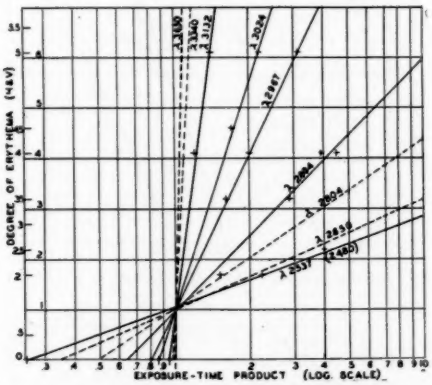

FIG. 3. *Erythema as a function of relative exposure. Except for ordinate —0— and the estimated half-units of the scale, the scale of ordinates measured from a point 0.7 units below 0, is proportional to the square root of the concentration of dye in Hausser and Vahle's permanent standards. Experimental points are indicated by crosses; estimated slopes by dotted lines.*

However, it was still possible that some simple function of the dye concentrations used by Hausser and Vahle would give a straight line relation. It was found by trial that if the square root of the dye concentration be plotted against the logarithm of the exposure producing the erythema, lines straight within the experimental error result, Fig. 3. The step from zero to one-half and all the other *estimated* half-steps, two to two and one-half, etc., were arbitrarily made equal to that from one-half to one. (As an analogy, in music the same interval is used in sharping the tones of the diatonic scale, in spite of the inequality of the dia-

tonic intervals). The significant point is that a single scale can be found which gives straight lines for all five wave lengths studied by Hausser and Vahle in this particular experiment.

The variation of the slope (*gamma*) of these lines with wave length is shown, on a logarithmic plot, in Fig. 4. Values of the slope for the three additional short wave lengths, 2480A, 2650A and 2804A, for which Hausser and Vahle give erythema data were obtained from Fig. 4 by interpolation or very slight extrapolation, and corresponding lines dotted in on Fig. 3. For the long wave lengths, 3342A and 3650A, Hausser and Vahle give only qualitative results, moreover the slope is already so

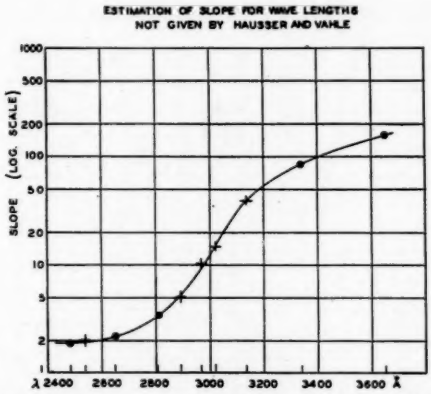

FIG. 4. *Slope of the lines in Fig. 3 as a function of wave length. Slopes taken from Fig. 3 are indicated by crosses; estimated slopes by dots.*

large that taking it as infinite would not greatly change the calculated exposure for any degree of erythema. It has therefore been deemed sufficient to extrapolate the curve of Fig. 4 on the assumption that the slope ('gamma') varies linearly with frequency (reciprocal wave length). Lines for these wave lengths have also been dotted in on Fig. 3.

In photographic work, it is found that the density varies with the time of development. In a somewhat analogous way, in the reddening of the skin by ultraviolet light, the degree of erythema varies with the time after exposure, rising to a maximum and then subsiding, with or without pigmentation. With the skin, unlike the photographic plate, this effect appears to vary markedly with the wave length, but with the limited data availabe it is not easy to dissociate this effect from the increase in duration with increasing degree of erythema. It is therefore

necessary to state at what interval after exposure the reddening is to be observed.

The curve given by Hausser and Vahle[2] for an interval of six and one-half hours has been taken as typical. These points,—together with approximate information given in their text for wave lengths 3132A, 3342A and 3650A,—have been combined with the slopes of Fig. 3, to give, Fig. 5, lines on a comparable energy basis. It will be seen that the short wave lengths 2480A and 2537A are relatively very effective for low degrees of erythema, losing this advantage for higher degrees. For the longer wave lengths, 3342A and 3650A, on the contrary, the energy required to produce minimum perceptible reddening needs to be increased only slightly to give a high degree of erythema.

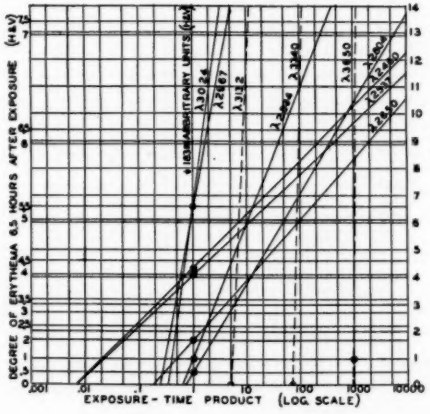

FIG. 5. *Erythema as a function of exposure. The unit of exposure (It=183 in Hausser and Vahle's arbitrary units) is of the order of one calorie per square decimeter.*

In order not to be limited to mercury radiation, it is necessary to express the erythemal effectiveness of radiation as a continuous function of the wave length. This has been done in Fig. 6, by drawing smooth curves through points read from Fig. 5, for five degrees of erythema: those designated by Hausser and Vahle[2] as degree 0 (threshold), one, three, four and five. Curves for other degrees of erythema could be drawn in a similar way.

The foregoing results apply strictly only to the effects of monochromatic radiation. No quantitative experimental work appears to have been done on the effect of combinations of two or a small number of different wave lengths; and the results with very complex mixtures of radi-

ation do not readily lend themselves to analysis. In default of experimental knowledge, it has been thought best to assume that the 'addition theorem' approximately valid in photographic work will also hold for erythema; this is tantamount to assuming that only one species of photochemical effect is produced by radiation of any wave length. There is some evidence of a qualitative difference in the erythemata *of the same*

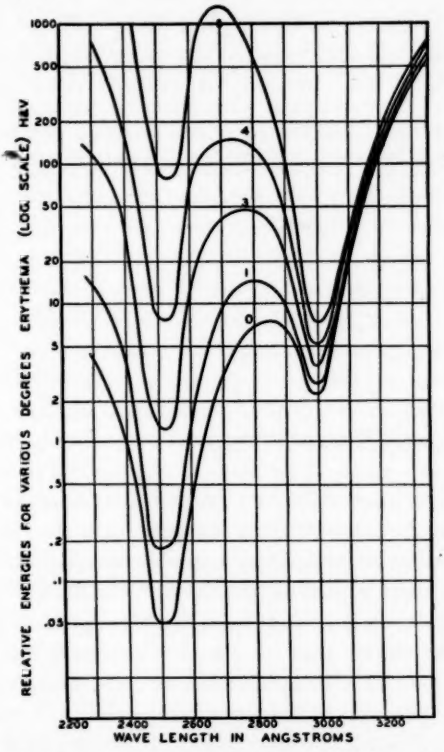

FIG. 6. *Energy of radiation at various wave lengths to produce erythema of degrees 0, 1, 3, 4 and 5 H & V. Exposure is in the same units as in Fig. 5.*

degree produced by radiations of different wave length, but it is not yet possible to take a quantitative account of this. The addition theorem referred to states that to produce any given density (degree of erythema) the ratios of the intensity at each wave length to the intensity necessary to produce the given density (degree of erythema)—when that wave length is acting alone—must sum up to unity.

Making use of curves of Figs. 5 and 6, showing the erythemal effectiveness of radiation of different wave lengths, that have been obtained from Hausser & Vahle's work together with the assumptions that have been discussed, the effectiveness of the radiation from a number of sources in producing various degrees of erythema has been calculated.

For a line spectrum the relative effectiveness of each line in producing a given degree of erythema may be obtained by dividing the energy flux density of the radiation comprising the line by the relative amount of energy of that wave length required to produce the given degree of erythema. (See Figs. 5 & 6). In Table 1 the energy flux density of the lines

TABLE 1. *Energy flux density and percentage of total erythematic effect for lines radiated by quartz mercury arc (at 72 volts and 3.75 amps) no reflector.*

Wave length		2537	2650	2700	2804	2894	2925	2967	3024	3130	3342	3650
$\mu w/cm^2$ at one meter distance		16.0	8.7	2.8	4.3	2.2	1.1	5.3	13.4	28.2	6.5	50.4
Percentage of total erythematic effect	threshold	95.	1.8	.3	.2	.1	.1	.8	1.3	.2	.003	.002
	degree 1	91.	2.0	.3	.3	.2	.2	2.1	3.5	.6	.01	.005
	degree 3	69.	1.9	.4	.6	.6	.7	8.1	15.7	3.0	.05	.03
	degree 4	37.	1.2	.3	.6	1.0	1.2	16.5	35.	7.6	.14	.09
	degree 5	7.4	.3	.1	.3	.8	1.3	22.	53.	14.1	.29	.17
Minimum perceptible	"A"	11.6	7.1	2.6	5.1	4.9	3.5	24.	35.	6.4	0	0
	"B"	47.	9.6	1.6	.6	.8	1.0	19.5	17.3	2.6	0	0

A—Hausser & Vahle, Strahlentherapie, *13*, 41; 1922.
B—Luckiesh, Holladay & Taylor, J.O.S.A., *20*, 423; 1930.

radiated from the quartz mercury arc (without reflector) as measured by Benford[5] and the corresponding percentages of the total erythematic effect to be ascribed to each line are given. These percentages are not strictly correct because some of the less intense lines which are not included in the table, such as the one at 2480A, may have an appreciable effect. This table shows that for the threshold and the first and third degrees of erythema the line at 2537A is responsible for most of the effect, but that for reddening of the fourth and fifth degrees the lines at 2967 and 3024A produce the major part of the action. For first degree erythema none of the lines other than those at 3024, 2967 and 2537 are of importance. The lines at 3130A are responsible for 3.0% of the effect at the third degree and for still higher percentages for more intense reddening.

For the sake of comparison two sets of data on the relative effectiveness of radiation of different wave lengths in producing "minimum perceptible erythema" have also been applied to this same quartz mercury arc data. The next to the last line of Table 1 shows the percentage of

the total effect to be ascribed to each line according to Hausser & Vahle's first article[1] on this subject. Judging from the proportion of the total effect ascribed to the line at 2537A it appears that the reddening designated as "just perceptible" in the first Hausser and Vahle article was slightly less intense than the fifth degree erythema of their second article. However the effectiveness of each line between 2537 and 2967A computed from the data of the first article is much greater than that for degree five of the second article. A little more stray radiation from the lines at 2537, 2967 and 3024A in the case of the measurements for the 1922 article would cause a discrepancy of this sort. Possibly more precautions were taken in the later measurements to eliminate stray radiation. Some of the differences such as that at 3130A may be due to observation of the erythema at a different time after exposure. No statement as to what this time was is given in the 1922 article.

The last line of Table 1 gives the corresponding percentages of the total erythematic effect computed from the erythemal effectiveness data published by Luckiesh,[6] Holladay and Taylor. Their "minimum perceptible erythema" apparently corresponds roughly with the fourth degree erythema of Hausser and Vahle's second article. However the percentage of the total effect obtained for the lines at 3130A is somewhat less than computed for degree three erythema, while that for the line at 2967A corresponds more nearly to degree 4.5. These differences may be due to Luckiesh, Holladay and Taylor's observations having been made twenty-four hours after the exposure while Figs. 5 & 6 were plotted from Hausser and Vahle's observations of erythema after a period of six and one-half hours. There is a relatively greater and less easily explained discrepancy between the two sets of data in the case of the lines at 2650 and 2700A. Since the latter is relatively weak, its erythemal effectiveness is difficult to determine by a filter method. For the line at 2650A, on the other hand, much better agreement would be expected, especially since fairly concordant results are obtained on all the lines between 2700 and 2967A, some of which are rather weak.

With a filter for eliminating most of the radiation of wave lengths shorter than 2700A, such as is used in the Uviarc Sunlamp, the lines at 2967 and 3024A are responsible for the major part of the effect, even for the threshold of erythema. Table 2 gives energy flux density values for the different lines as radiated from the arc with filter but without a re-

[5] F. Benford, J. Mot. Pic. Eng. *14*, 409; 1930. Mr. Benford has very kindly furnished us the energy flux density values for the individual lines listed in Tables 1 and 2.

[6] M. Luckiesh, L. L. Holladay & A. H. Taylor, J.O.S.A. *20*, 423; 1930.

flector and the corresponding percentages of the total erythematic effect. The intensities of the lines were obtained by multiplying Benford's[5] values for the output of the bare arc at each wave length by the transmission of the Uviarc filter as given by Buttolph.[7]

In the case of the Type S-1 lamp which is the source of radiation in the General Electric Sunlamp, the bulb acts as a filter in reducing the intensity of the radiation of wave lengths shorter than 2900A. Data on

TABLE 2. *Energy flux density and percent of total erythematic effect for each line for Uviarc with filter but without reflector. (At 72 volts and 3.75 amperes).*

Wave length		2700	2804	2894	2925	2967	3024	3130	3342	3650
$\mu w/cm^2$ at 1 meter distance		.04	.76	.96	.56	3.14	9.2	22.6	5.7	45.3
Percentage of total erythematic effect	threshold	.23	2.2	3.1	2.5	29.	53.5	9.3	.16	.09
	degree 1	.11	1.3	2.3	2.4	28.	55.	10.6	.19	.11
	degree 4	.01	.26	1.0	1.4	24.	58.	14.7	.30	.19

the energy radiated in each of the principal ultraviolet lines in the case of both the bare Type S-1 lamp and the complete Sunlamp unit were published[8] recently. The energy flux density in the center of the Sunlamp beam at one meter distance from the lamp and the computed percent of the total erythematic effect are given for each ultraviolet line of

TABLE 3. *Energy flux density and percent of total erythematic effect for each line in radiation in center of beam from General Electric Sunlamp.*
(115 Volts on Primary)

Wave length		2537	2650	2804	2894	2967	3024	3130
$\mu w/cm^2$ at one meter distance		.32	1.9	2.9	3.3	12.	23.	67.
Percentage of total erythematic effect	threshold	27.	5.4	1.9	2.5	26.	31.	6.
	degree 1	12.	2.8	1.5	2.3	32.	40.	9.
	degree 4	0.6	0.2	0.3	1.2	32.	51.	15.

importance in Table 3. It will be noted that the line at 2537A has quite an appreciable effect in spite of its low intensity, in the case of threshold or first degree erythema. However, the lines in the neighborhood of 3000A produced the major part of the effect in each case. Just as with the quartz mercury arc, the lines at 3024A are more effective than the one at 2967A. The lines at 3130A have considerable effect—even in producing the threshold of erythema—but much less than the one at 2967A.

[7] L. J. Buttolph, G. E. Vapor Lamp Co., Bulletin #111.

[8] B. T. Barnes, Phys. Rev. *36*, 1473; 1930. W. E. Forsythe, B. T. Barnes & M. A. Easley, J.O.S.A. *21*, 34; 1931.

The lines at 3342 and 3650A,—although they are strong,—are practically ineffective for erythema production.

For a continuous spectrum the relative erythemal effectiveness of the radiation in different wave-length regions is obtained by a procedure analogous to that used with a line spectrum. In the case of heated tungsten the spectral distribution of the radiation in the portion of the ultraviolet in which we are interested may be computed by making a rel-

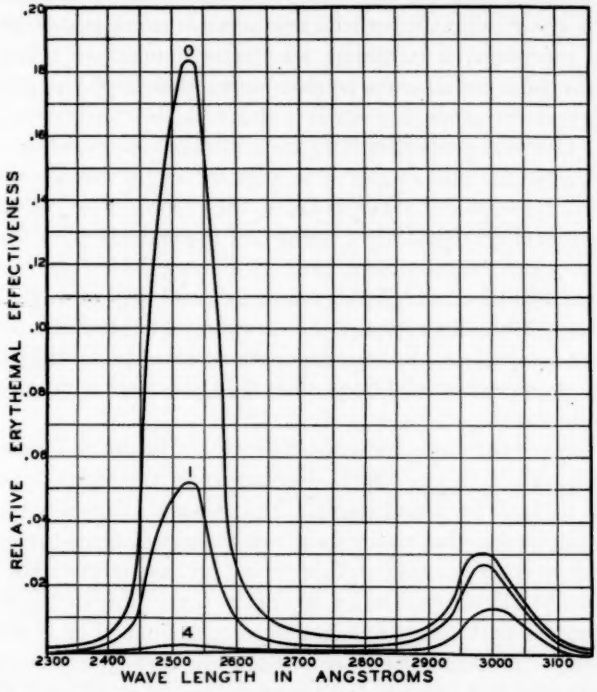

FIG. 7. *Effectiveness per 10A in producing erythema of degrees 0 (threshold), 1 and 4 for radiation from a tungsten filament at 3040°K in a quartz bulb. Total effect for degree 1 taken as unity.*

atively slight extrapolation of the spectral emissivity curve[9] obtained by plotting the data of various investigators. Curves for the erythemal effectiveness of the radiation from tungsten at 3040°K in a thin fused quartz bulb are given by Fig. 7. The ordinate gives the relative effectiveness in producing a given degree of reddening for a 10A band at any wave length, taking the effectiveness of the total radiation of all wave lengths in producing erythema of degree one as unity. It is evident that

the radiation of wave lengths between 2450 and 2600A contributes most toward bringing the skin to the state where it is at the threshold of erythema but that radiation of wave lengths greater than 2900A has by far the greatest effect in producing fourth degree erythema. For the threshold 82% of the effect is to be ascribed to radiation of wave lengths shorter than 2800A; for first degree erythema 63% and for fourth degree only 7.4% of the total action is due to this wave-length region. These curves show that with a tungsten source use of a quartz bulb is a decided advantage if a very slight superficial erythematic effect is desired. For producing more intense reddening a tungsten filament in a bulb with high transmission for all wave lengths longer than 2900A is almost as satisfactory as the same filament in a quartz bulb. The exposures required are inversely proportional to the area under the curves in Fig. 7. With a tungsten filament at 3040°K in a quartz bulb if the exposure necessary for the threshold is taken as unity, the exposures for the first and fourth degrees of erythema are 2.6 and 12.9 respectively. For the same filament in a bulb transmitting most of the energy for wave lengths longer than 2900A but a negligible amount of radiation of wave lengths shorter than 2700A, the ratios of the exposures for threshold, first and fourth degree erythema are approximately as 1.0: 1.3: 2.5, but the exposure for the threshold will be at least 5.6 times that required with a quartz bulb.

In the case of the sun's radiation there is no measurable energy of wave lengths shorter than 2900A. Consequently there can be but one maximum in each erythema curve. The spectral distribution of the ultraviolet radiation that reaches the earth's surface from the sun has been obtained[9] from the data of Abbot[10] and his associates and of Fabry and Buisson.[11] The corresponding erythemal action due to radiation in different wave-length bands has then been computed. Fig. 8 gives the effectiveness of the sun's radiation through one atmosphere in producing the threshold and the first and fourth degrees of erythema. The ordinate is the relative effectiveness per 10A taking the effectiveness of the whole spectrum in producing first degree erythema as unity. The maximum for each curve in Fig. 8 comes at about 3060A and the ordinate dividing the area under the curve into halves is located in the region between 3100 and 3130A. Thus nearly one-half the erythematic effect of the sun is due to energy in the wave-length region longer than

[9] W. E. Forsythe & F. L. Christison, J.O.S.A. *20*, 396; 1930.
[10] Smithsonian Misc. Collection, *74*, 15; 1923. Annals Astrophys. Obs. *2*, 112; 1908.
[11] Comptes Rendus *175*, 156; 1922. Astrophys. Jour. *54*, 297; 1921.

3130A where the radiation from tungsten or from a mercury arc is relatively ineffective. This difference is due to the extreme steepness of the sun's spectral energy curve between 3000 and 3400A.

The curve for the relative effectiveness in producing erythema of degree one of the radiation from the sun transmitted by a piece of window glass (spectral transmission, Fig. 9) 46 mils thick is also given in Fig. 8, Curve *A*. It will be noted that the maximum effectiveness comes at 3350A and is only one-twentieth of the maximum effectiveness obtained with direct radiation.

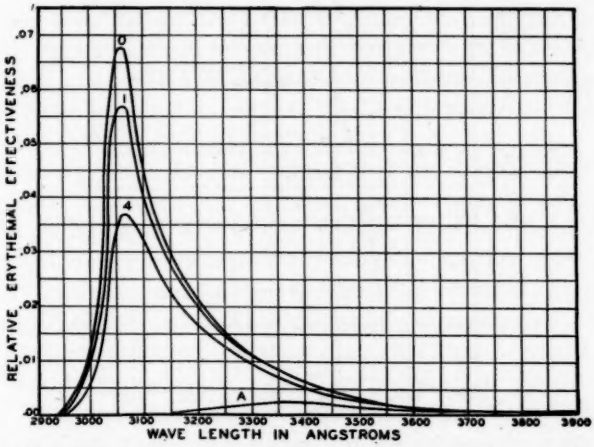

FIG. 8. *Spectral distribution of effectiveness of sun's radiation through one atmosphere (sun directly overhead) in producing erythema of degrees 0 (threshold), 1 and 4 and also (Curve A) degree 1 after transmission through window glass 46 mils thick. Scale: effectiveness per 10A taking total effectiveness (area under curve) for degree 1 as unity.*

The results that have been given show the relative effectiveness of different spectral regions of the radiation from each source in producing a given degree of erythema. It is interesting to compare the relative effectiveness of the total radiation of these different sources in producing various degrees of erythema.

For convenient comparison the length of the exposure to sunlight transmitted through one atmosphere necessary to produce reddening of degree one has been taken as a unit for computing the corresponding exposure times for other sources at this and other degrees of erythema. Table 4 shows the results of this computation. The figures for the artificial

sources all refer to the radiation at one meter distance and for the operating conditions given in the table.

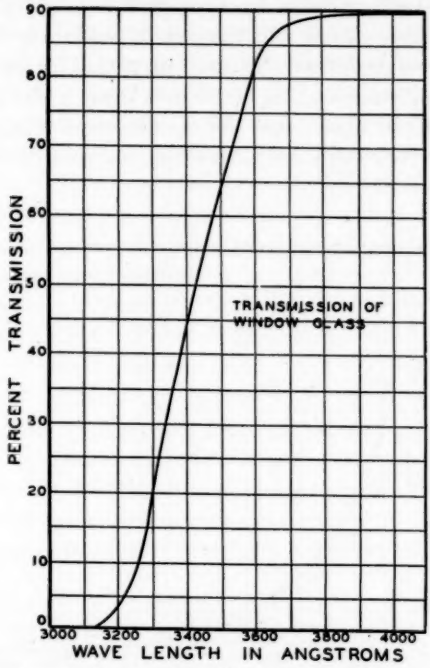

FIG. 9. *Transmission curve for a sample of window glass 46 mils thick.*

TABLE 4. *Relative lengths of exposure necessary to produce a given degree of reddening with various sources. (Subject at one meter distance from each artificial source).*

Degree of reddening	Sun through one atm	Sun through one atm window glass 46 mils thick	Uviarc* without filter	Uviarc* with filter	Type† S—1 lamp	G. E.‡ Sunlamp	500 W§ lamp with quartz bulb	500 W§ lamp with 1 mm. corex D bulb
0	.90	—	.020	1.22	1.34	.28	10.	47
1	1.00	22	.066	1.50	2.25	.44	25.	67
4	1.31	—	1.07	2.62	5.0	.90	120.	140

* No reflector. Arc at 72.5 volts and 3.75 amperes.

† In direction perpendicular to plane of leads. 115 volts on primary of transformer. No reflector.

‡ In center of beam. Reflector pointing downward. 115 volts on primary.

§ No reflector. Tungsten filament at 3040°K. (300 hrs. life).

These results show that for a 272 watt quartz mercury arc the exposure time required for fourth degree erythema would be 82% of that required by the sun's radiation through one atmosphere. For producing the threshold or first degree of erythema the quartz mercury arc without filter is very effective on account of the large amount of short wave-length radiation present. Under the conditions stated the exposure for the threshold of erythema is only 1/45 as great for the quartz mercury arc as for the sun. However, for easily perceptible reddening exposures more nearly comparable would be required. Erythema would probably escape casual observation unless more intense than the first degree.

With the filter used in the Uviarc Sunlamp the exposure necessary for fourth degree erythema appears to be more than twice that without the filter; for threshold and first degree erythema the exposure must be many times greater than without the filter on account of the elimination of the short wave-length radiation.

The figures given in Table 4 indicate that the General Electric Sunlamp requires approximately the same exposure time for fourth degree erythema as the quartz mercury arc without filter or reflector, but relatively much longer exposures for the threshold and first degree of erythema—due to the reduction by the bulb of the intensity of the short wave-length radiation. The computed lengths of exposures for the General Electric Sunlamp are less than one-half those calculated for the quartz mercury arc with filter but without reflector. It should be noted, however, that a suitable reflector such as is used with the Uviarc Sunlamp probably increases the intensity of the ultraviolet radiation received at one meter distance from the lamp by more than a factor of two and correspondingly reduces the exposure time necessary for each degree of erythema. The Uviarc Sunlamp can therefore be expected to produce each degree of erythema with a length of exposure of the same order of magnitude as required for the General Electric Sunlamp. The unfiltered radiation from a quartz mercury arc in a suitable reflector would produce erythema in a considerably shorter time than that from either of these Sunlamp sources, but is, of course, less desirable for home use on account of the necessity of protecting the eyes during exposures. (A certain degree of caution is necessary with regard to exposure of the eyes to any ultraviolet source. The face should not be exposed to radiation from the General Electric Sunlamp long enough to produce erythema unless the eyelids are closed tightly or the eyes are otherwise protected.)

A 500 watt lamp with a tungsten filament at 3040°K and a quartz bulb

requires a much longer exposure for producing erythema than any of the other sources for which computed relative exposure times are given in Table 4 (except for radiation from the sun transmitted by window glass). If the bulb were of a high grade of ultraviolet transmitting glass such as Corex D, instead of quartz, the required exposure time for fourth degree erythema would be approximately 1.2 times as long. The exposure for the threshold of erythema would be 4.7 times and for the first degree 2.7 times that for a similar lamp with a quartz bulb.

It is to be borne in mind that the foregoing calculations depend at many points on very considerable extrapolations from the experimental data of Hausser and Vahle. The dependability of the results of the calculations is therefore contingent not only upon the accuracy of the measurements of Hausser and Vahle, but upon the assumptions used in the reduction of the data—in particular the tacit assumption that the immediate photochemical reactions produced by radiations of various wave lengths are the same, and therefore additive. It must likewise be remembered that the Hausser and Vahle results are stated for a normal 'unpigmented skin' and that the result in a given case will depend upon the individual, upon the part of the body exposed, and upon previous exposure to erythematogenous radiation. Until the effect of these three factors has been reduced to a quantitative basis, no claims of high precision in any calculations of erythemal effect will be warranted. Such a reduction will require a considerable volume of additional experimental data.

THERMOLUMINESCENCE EXCITED BY EXPOSURE TO RADIUM

By Frances G. Wick

[Vassar College, Poughkeepsie, N. Y. Received January 12, 1931]

The thermoluminescence excited in certain natural crystals and synthetic substances by exposure to x-rays and cathode rays has been the subject of previous investigations.[1,2,3,4] Through the courtesy of Dr. Stefan Meyer and Dr. Karl Przibram the opportunity was given the author of making a study[5] of some of the same materials after exposure to radium at the Institute for Radium Research in Vienna. Observations were made upon calcium sulphate containing a small amount of manganese and upon specimens of fluorite from Westmoreland, New Hampshire and from Weardale, England. The effect of subjecting these materials to high pressure before and after exposure to radium was studied at the suggestion of Dr. Przibram.

That phosphorescent materials are greatly changed by subjection to pressure has been shown by Lenard.[6] It has been found by Przibram[7] and his associates that natural and artificial crystals which are made thermoluminescent by exposure to radium have the thermoluminescence resulting from a given exposure greatly altered by subjecting the crystals to pressure either before or after irriadiation. Przibram[7] has also done extended work upon the coloration of crystals by exposure to radium and the changes in color produced by subjecting materials to pressure.

Method of Observation

The materials used for this work were powdered in a mortar, sifted through a gauze of mesh 100 to the inch and put into a bronze holder with a circular depression in it 6 mm in diameter and 1 mm deep. The specimen, covered by a thin window of mica, was exposed at a distance of 2 mm to a preparation of radium chloride containing 94 mg of radium element mounted in a glass tube 3.5 cm long, 4 mm in diameter and,

[1] Wick, J.O.S.A. & R.S.I., *14*, 33; 1927.

[2] Wick and Slattery, J.O.S.A. & R.S.I., *14*, 125; 1927.

[3] Wick and Slattery, J.O.S.A. & R.S.I., *16*, 398; 1928.

[4] Wick and Carter, J.O.S.A. & R.S.I., *18*, 383; 1929.

[5] Wick, Mitt. d. Inst. f. Ra-Forsch., Nr 265..

[6] P. Lenard, Handbuch der Experimentalphysik von Wien und Harms, Bd. XXIII/I, p. 627 u.f.

[7] K. Przibram, Mitt. d. Inst. f. Ra-Forsch., Nr. 196, Nr. 206; Wr. Ber. IIa, 136, 43, 435, 1927. Mitt. Nr. 230, Wr. Ber. IIIa, 138, 230, 1929.

.25 mm thick. All specimens were exposed for 5 minutes unless otherwise stated.

Five minutes after the end of the exposure the thermoluminescence emitted when the exposed materials were heated was measured by a method used by Przibram and perfected by Urbach.[8] Apparatus which they had set up was generously put at the disposal of the author. The specimen, still mounted in the holder, was placed in the center of a small electric oven at an angle of 45 degrees in front of a Günther and Tegetmeyer rubidium photoelectric cell which was connected with a Wulf einfaden electrometer. The energy emitted as visible light as the specimen warmed up and became thermoluminescent was indicated by the deflections of the electrometer indicator. Readings were made of this deflection and, simultaneously, the time from the beginning of the heating was counted by the beats of a metronome. Observations were made until the specimen, having passed through one or more maxima, ceased to emit light or emitted it at a very low and practically constant rate. Since specimens, after exposure to radium, are known[9] to be very sensitive to light all the operations were carried on in the dark.

CALCIUM SULPHATE PLUS MANGANESE

The calcium sulphate plus manganese used was prepared by the method described by Wiedemann and Schmidt[10] and contained one atom of manganese to every 125 molecules of calcium sulphate. After exposure to radium it has a bright green thermoluminescence which comes up at a relatively low temperature in a band which has the approximate limits of 4800 to 5600A. Material which has been compressed to 10,000 kg/cm^2 before exposure to radium emits a thermoluminescence which comes up at the same temperature but is much less intense than that excited in an uncompressed specimen. The results of similar exposure to radium of the uncompressed and compressed material are shown in Fig. 1. Electrometer deflections indicating relative brightness are plotted against the time from the beginning of heating the specimens, the temperature being indicated by the dotted line. The material compressed before irradiation emits much less energy as shown in curve 2 than the uncompressed material shown in curve 1. A pres-

[8] I. Urbach, Mitt. d. Inst. f. Ra-Forsch., Nr. 262.

[9] K. Przibram and E. Kara-Michailova, Mitt. d. Inst. f. Ra-Forsch. Nr. 149 Wr. Ber. IIa, *131*, 40; 1922.

[10] Wiedemann and G. C. Schmidt, Ann. d. Phys., *56*, 201; 1895.

sure of 10,000 kg/cm² produced an effect which persisted without change in material which was allowed to stand for 21 days. A pressure of 5000 kg/cm² produced an effect considerably less than that of 10,000 kg/cm² and after 16 days most of the effect had disappeared. A pressure of 1000 kg/cm² produced no effect.

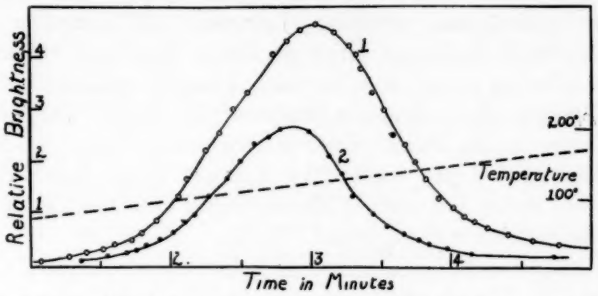

FIG. 1. *Calcium sulphate plus manganese. Effect of pressure before exposure to radium. 1—uncompressed. 2—compressed.*

The effect of compressing the material to 10,000 kg/cm² after exposure to radium is to reduce the thermoluminescence upon subsequent heating as shown in Fig. 2. The difference in the emission of the compressed and uncompressed material is not so great as is the case in material compressed before exposure to radium.

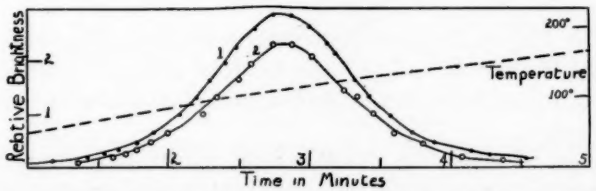

FIG. 2. *Calcium sulphate plus manganese. Effect of pressure after exposure to radium. 1—uncompressed. 2—compressed.*

Exposure of calcium sulphate plus manganese to light after exposure to radium reduces the thermoluminescence upon subsequent heating. The thermoluminescence of a specimen exposed to radium for 11 minutes was entirely quenched by exposure to the noonday sun of Vienna in April for 45 minutes. Sunlight through a blue filter has a strong quenching effect; through a red filter very little effect.

FLUORITE FROM WEARDALE, ENGLAND

Many specimens of fluorite from different localities have a natural thermoluminescence the spectra of which contain narrow bands due to the presence of rare earth impurities as shown[11] by a spectroscopic study of the luminescence excited in them by cathode rays.

Blue-green fluorite from Weardale, England containing as impurities samarium, gladolinium, terbium, dysprosium and europium has a natural thermoluminescence which persists a long time. When this specimen is heated rapidly on a hot plate it shows a succession of colors changing rapidly and ending in a persistent blue-violet. The spectrum shows narrow bands characteristic of rare earths coming up in succession which are finally masked by a broad, diffuse band which is probably not due to rare earths. Measurements of this natural thermoluminescence are shown in Curve 1, Fig. 3.

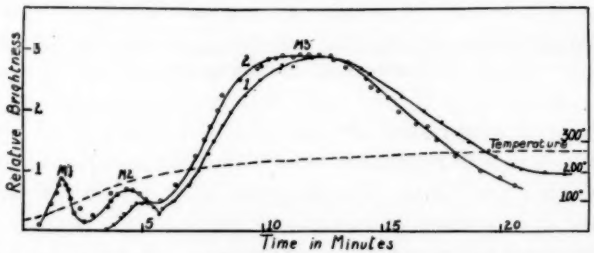

FIG. 3. *Fluorite from Weardale. Effect of exposure to radium upon natural thermoluminescence. 1—unexposed. 2—exposed.*

Curve 2, Fig. 3, shows measurements made upon the natural material after exposure to radium. The spectrum shows an intensification of the bright lines due to rare earths but no evident change in the broad, diffuse band.

The effect of subjecting this Weardale fluorite to pressure was to greatly reduce the intensity of the natural thermoluminescence as shown in Fig. 4. Curve 1 shows that the intensity of light emitted by the uncompressed natural material is much greater than that emitted by a similar specimen after subjection to a pressure of 10,000 kg/cm² shown in curve 2.

Some of this same Weardale fluorite was heated to remove all natural thermoluminescence then exposed to radium. Measurements of the resulting thermoluminescence are shown in Fig. 5, curve 1. Comparison

[11] Wick, Phys. Rev., *24*, 272; 1924.

of this with curve 2, Fig. 3 shows that the first maximum is much higher in the heated specimen, that the second maximum is about equally intense in both and that the broad, diffuse band is much more intense in the unheated specimen probably masking less intense maxima which

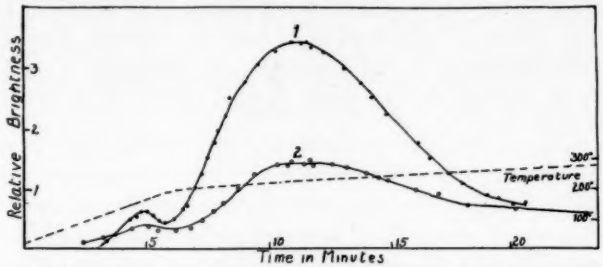

FIG. 4. *Fluorite from Weardale. Effect of pressure upon the natural thermoluminescence. 1—uncompressed. 2—compressed.*

are brought out in the heated specimen. The sharp bands are intensified by heating; the broad, diffuse band present in the natural material is not strongly excited by a short exposure to radium.

The effect of pressure before exposure to radium upon this heated material is shown in Fig. 5. The maxima are all lower in the compressed

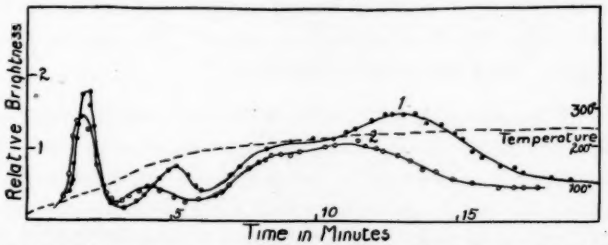

FIG. 5. *Fluorite from Weardale previously heated to remove natural thermoluminescence. Effect of pressure before exposure to radium. 1—uncompressed. 2—compressed.*

material shown in curve 2 than in the uncompressed shown in curve 1. These maxima are probably due, in large part, to rare earths.

The effect of pressure after exposure to radium is shown in Fig. 6. The thermoluminescence is reduced and the sharp maxima due to the rare earths have almost disappeared.

FLUORITE FROM NEW HAMPSHIRE

Transparent green fluorite from Westmoreland, New Hampshire has an exceptionally brilliant natural thermoluminescence the color of which

changes several times during heating. Spectroscopic observations show, at relatively low temperatures, sharp line-like bands due to rare earths and, at higher temperatures, a broad, diffuse band in the blue end of the spectrum. The rare earth bands are much brighter after exposure to radium and, as the exposed specimen is warmed up, bands due to different rare earths are observed to appear at different temperatures. The relative brightness of the bands changes markedly as the heating continues accounting for the color changes observed in the thermoluminescence.

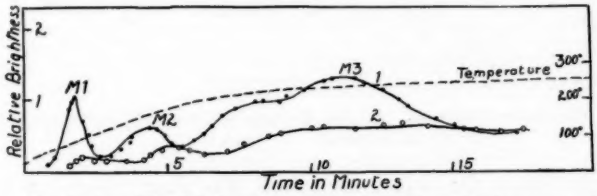

FIG. 6. *Fluorite from Weardale heated to remove natural thermoluminescence. Effect of pressure after exposure to radium. 1—uncompressed. 2—compressed.*

The thermoluminescence in the natural specimen was too intense for measurement upon the apparatus as adjusted for this work. Measurements were made upon material heated until all natural thermoluminescence was exhausted then exposed to radium. The results, shown in Fig. 7, curve 1, show four distinct bands.

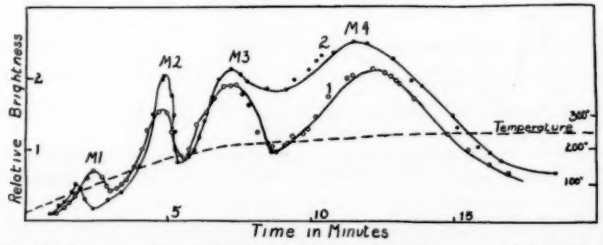

FIG. 7. *Fluorite from New Hampshire, previously heated to remove natural thermoluminescence. Effect of pressure before exposure to radium. 1—uncompressed. 2—compressed.*

An attempt was made to follow visually the changes in the thermoluminescence spectrum as the specimen was heated in the oven used in front of the photoelectric cell but the intensity of light in material heated so slowly was not great enough for such observations. It was possible, however, to observe the changes in the spectrum of material heated

quickly by placing it in a copper holder upon a plate which was already at the maximum temperature attained by the slower heating used for Curve 1. During this rapid heating certain rare earth bands were observed to reach a maximum brightness in succession appearing to correspond to the four maxima of Curve 1. These same rare earth bands were observed more easily in a specimen of unpulverized crystal coming from the same locality. Observations upon the spectra of specimens of this crystal, some previously heated and some natural, exposed to radium for 22 hours showed that certain bands brightened up and dimmed in succession: a green band not present in the crystal which had not been exposed to radium came up at a very low temperature due probably to samarium, then came a yellow band due to dysprosium, then two green bands due to terbium. The approximate wave lengths of these bands were observed with a direct vision spectrometer of Kovalski and Hower. Comparison with the corresponding bands observed by Urbain[12] in the cathode ray spectra of rare earths in calcium fluoride is shown in the following table.

Maximum	*Wave length in A*	*Element*	*Wave length of Urbain*[12]
1	5700	Samarium	568–565
2	5800–5900	Dysprosium	585–582
3	5550	Terbium	552.5
4	5380	Terbium	540, 541

A band between 4800 and 5000 with a sharp maximum at 5000 also probably due to terbium was observed at the same time as maxima 2, 3, and 4. A much broader and more diffuse band in the blue end of the spectrum appeared when the temperature was made still higher and it persisted until red heat was reached. The presence of this band is indicated by the fact shown in the curve that the intensity of luminescence is not reduced to zero after the four maxima due to rare earths have disappeared.

The effect of subjecting this material to 10,000 kg/cm² before exposure to radium is shown in Curve 2, Fig. 7. It will be observed that the total amount of energy emitted by the compressed specimen is greater than that by the uncompressed. In the compressed specimen the diffuse blue band which, in the uncompressed specimen becomes prominent only at higher temperatures than those used comes out strongly at a lower temperature. Maxima M_3 and M_4 are less sharply defined in the com-

[12] G. Urbain, Ann. de Chim et de Phys. *18*, 562; 1009.

pressed material. It appears that the effect of pressure is to diminish the brightness of these rare earth bands and to bring out at a lower temperature the broad diffuse band due to some other impurity. The luminescence centers involving the rare earths may be different in character from those with which the other impurities are connected. Pressure increases the amount of energy absorbed by one upon exposure to radium and decreases the amount absorbed by the other.

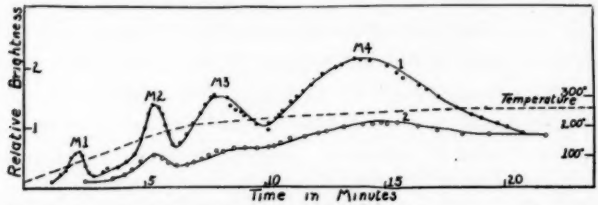

FIG. 8. *Fluorite from New Hampshire previously heated to remove natural thermoluminescence. Effect of pressure after exposure to radium. 1—Uncompressed. 2—Compressed.*

The effect of pressure of 10,000 kg/cm² after exposure to radium is shown in Fig. 8. A comparison of Curve 1, taken from the uncompressed material, with Curve 2 from the compressed material shows that the sharp bands due to the rare earths are practically removed by the pressure. The intensity of the diffuse band is not reduced to the same extent.

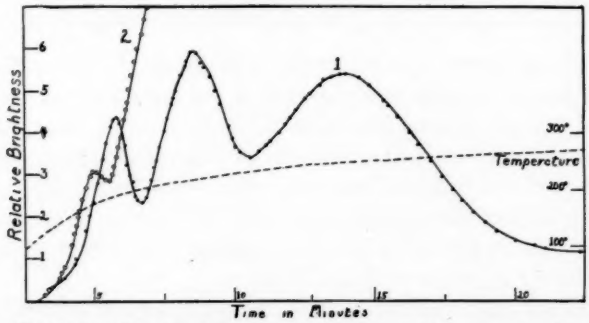

FIG. 9. *Fluorite from New Hampshire. 1—Specimen heated to remove natural thermoluminescence then exposed to x-rays. Observations made 10 months after exposure. 2—Natural thermoluminescence.*

Some of the same fluorite from New Hampshire heated to remove natural thermoluminescence and exposed to x-rays at a voltage of 75,000 and a current of 1.5 milliamperes for ten minutes ten months previously was measured with the photoelectric cell. The resulting

curve, Fig. 9, curve 1 shows three strong maxima of greater intensity than those due to an exposure to radium of five minutes an interval of only five minutes elapsing between the time of exposure and observation. The maxima come up at about the same temperatures as those in the material exposed to radium. Curve 2, Fig. 9 shows an attempt to measure the thermoluminescence of a natural specimen of the same material which had not been previously heated or exposed to any radiation. The intensity is too great for the range of the measuring outfit.

The effects produced in these materials by exposure to radium are similar to those produced by x-rays. As has previously been observed[3] exposure of crystals of fluorite and calcite to x-rays results in a thermoluminescence which, upon heating the specimen, comes up quickly and lasts for a very short time, followed by one or more other maxima which come up more slowly and last longer. Visual observations of crystals observed after exposure to x-rays were made after exposure to radium. The same maxima appeared and the same colors of luminescence—possibly the first maximum was more pronounced than in the case of exposure to x-rays.

After exposure of the crystals to high voltage cathode rays only the more persistent luminescence was observed by the visual method used for this work[4] although there was in some cases a slight suggestion of a flash coming up quickly at a lower temperature. Professor Przibram suggested that the powerful bombardment of the cathode rays may have caused the emission of the first maximum during exposure. These materials were intensely luminescent during exposure to cathode rays.

The author wishes to express her sincere appreciation of the many suggestions made by Dr. Przibram during the whole course of this work and to Dr. Meyer for putting at her disposal the fine facilities of the Radium Institute in a most generous way. She wishes to gratefully acknowledge the efficient help of Mr. George Schwartz in the manipulation of the apparatus.

ON THE INDICES OF REFRACTION OF LIQUIDS

By Morton Masius and W. E. Lawton
[Worcester Polytechnic Institute, Worcester, Massachusetts.
Received January 14, 1931]

1. Introduction

The index of refraction of a liquid may be obtained by enclosing the liquid in a hollow prism, measuring A, the total external angle of the prism, and δ, the minimum deviation of the transmitted light, and calculating n, the index of refration, by the well known formula

$$n = \frac{\sin \frac{1}{2}(A - \delta)}{\sin \frac{1}{2}A}.$$

The signs of the angles appearing in this equation, as well as in all other equations of this paper, are chosen, as in our earlier publication on this subject,[1] according to the system proposed by J. Guild.[2] In our earlier publication we considered the effect produced on the result, when the glass covers of the hollow prism are not true optical plates but thin prisms of angles α and β respectively. The equation we obtained was

$$n = \frac{\sin \frac{1}{2}(A - \delta)}{\sin \frac{1}{2}A}\left[1 - \left(\frac{\alpha + \beta}{2}\right)\left(\cot z - \cot \frac{A}{2}\right)\right.$$
$$\left. - \left(\frac{\alpha + \beta}{2}\right)^2 \cot \frac{A}{2}\left(\cot z - \cot \frac{A}{2}\right) + \frac{1}{2}\left(\frac{\beta - \alpha}{2}\right)^2 \frac{(g^2 - 1)(g^2 - n^2)}{(n^2 - 1)g^2 \cos^2 z}\right]. \quad (1)$$

In this equation g is the index of refraction of the glass covers, A is the total external angle of the prism, and z is an auxiliary angle defined by

$$g \sin z = \sin \tfrac{1}{2}(A - \delta) \quad (2)$$

and the reduction from air to a vacuum has not yet been made.

We showed that in most cases the terms involving the higher powers of the small quantities α and β may be neglected and the equation used in the form

$$n = \frac{\sin \frac{1}{2}(A - \delta)}{\sin \frac{1}{2}A}\left[1 - \left(\frac{\alpha + \beta}{2}\right)\left(\cot z - \cot \frac{A}{2}\right)\right]. \quad (3)$$

The use of this equation in practice is not convenient, partly because A must be measured with the spectrometer with the same accuracy as δ,

[1] J.O.S.A., *20*, 271; 1930.

[2] Article on Spectroscopes and Refractometers in R. Glazebrook's *Dictionary of Applied Physics*.

which, as J. Guild[3] has shown, is difficult, and partly because the two other quantities required, *viz.* g and $(\alpha+\beta)$, though they need not be measured with great accuracy, require additional adjustments and methods or auxiliary apparatus.

We propose to show in this paper how the determination of n may be reduced to the measurement of *three similar* quantities, *viz.* three minimum deviations, instead of the *four dissimilar* quantities A, δ, g, and $(\alpha+\beta)$. Thereafter we shall point out how, when the values of n for a few liquids have been found accurately, the values of n for all other liquids can be found by an exact interpolation formula, the use of which requires only *one* measurement for each liquid.

2. USE OF THE AVERAGE OF THE THREE MINIMUM DEVIATIONS

In the case of a solid prism there is a method suggested by Glazebrook and carried out by Gifford,[4] by which the index of refraction of the solid is determined from $\bar{\delta}$, the average of the three minimum deviations corresponding to the three angles of the prism. It depends upon the fact that $\bar{\delta}$ is very nearly equal to the minimum deviation corresponding to 60° exactly. Hence, providing each of the three angles of the prism differs from 60° by less than ten minutes, the index of refraction may be calculated from

$$n = \frac{\sin \frac{1}{2}(60^\circ - \bar{\delta})}{\sin \frac{1}{2}(60^\circ)} = 2 \sin \frac{1}{2}(60^\circ - \bar{\delta})$$

without introducing an error of more than one unit of the sixth decimal. This obviates the necessity of measuring A. We shall now show that this same method may be used to determine the index of refraction of a liquid enclosed in a hollow prism.

In addition to the quantities already mentioned we shall use the quantities B, C, η, θ defined by the following equations:

$$\left.\begin{aligned} n \sin \tfrac{1}{2}(60^\circ) &= \sin B = g \sin C \\ A &= 60^\circ + \eta \\ A - \delta &= 2B + \eta - \theta. \end{aligned}\right\} \quad (4)$$

For brevity we shall put:

$$\left.\begin{aligned} \sigma &= \alpha + \beta \\ \tau &= \beta - \alpha \\ f &= \frac{(g^2 - 1)(g^2 - n^2)}{(n^2 - 1)g^2 \cos^2 C}. \end{aligned}\right\} \quad (5)$$

[3] *loc. cit.*

[4] Proc. Royal Soc. *70*, 330; 1902.

Equation (1) may now be written in the form

$$n = \frac{\sin \frac{1}{2}(2B + \eta - \theta)}{\sin \frac{1}{2}(60^\circ + \eta)}\left[1 - \frac{\sigma}{2}\left(\cot z - \cot\frac{A}{2}\right)\right.$$
$$\left. - \frac{\sigma^2}{4}\cot\frac{A}{2}\left(\cot z - \cot\frac{A}{2}\right) + \frac{\tau^2}{8}f\cos^2 C\left(1 + \frac{1}{\cot^2 z}\right)\right]. \quad (6)$$

In these equations the angles $\eta, \theta, \sigma, \tau$ are all small. In equations containing them we shall make use of the following approximations, where h is small compared with unity and ϵ is small compared with ϕ:

$$\frac{1}{1+h} = 1 - h + h^2 \text{ and } (1 + h_1)(1 + h_2) = 1 + h_1 + h_2 + h_1h_2 \quad (7)$$

$$\sin\epsilon = \epsilon \qquad \text{and} \qquad \cos\epsilon = 1 - \frac{\epsilon^2}{2} \quad (8)$$

from which we find

$$\sin(\phi + \epsilon) = \sin\phi\left(1 + \epsilon\cot\phi - \frac{\epsilon^2}{2}\right). \quad (9)$$

All these approximations are carried to the second power of the small quantities. In the case of certain terms, which would yield the third or higher powers of the small quantities, the approximations need not be carried further than the first power of the small quantities. Such first order approximations required in our calculations are:

$$\left.\begin{aligned} \cot\tfrac{1}{2}A &= \cot 30^\circ - 2\eta \\ \cot z &= \cot C\left[1 - \frac{1}{2}(\eta - \theta)\frac{\cot B}{\cos^2 C}\right]. \end{aligned}\right\} \quad (10)$$

From relations (9), (10), and (6) we have:

$$n = n\frac{\left[1 + \frac{1}{2}(\eta - \theta)\cot B - \frac{1}{8}(\eta - \theta)^2\right]}{\left[1 + \frac{1}{2}\eta\cot 30^\circ - \frac{1}{8}\eta^2\right]}\left[1 - \frac{\sigma}{2}(\cot C - \cot 30^\circ)\right.$$
$$\left. - \frac{\sigma^2}{4}\cot 30^\circ(\cot C - \cot 30^\circ) + \frac{\tau^2}{8}f - \eta\sigma + \frac{\sigma}{4}(\eta - \theta)\frac{\cot B\cot C}{\cos^2 C}\right]$$

which, when rearranged by use of (7), yields:

$$\theta = \eta(1 - \cot 30^\circ\tan B)\left[1 + \frac{\eta}{2}\tan B - \frac{\theta}{4}\tan B - \frac{\sigma}{2}\frac{\cot C}{\cos^2 C}\right]$$

$$-\sigma(\cot C-\cot 30^\circ)\tan B\left[1+\frac{\eta}{2}\tan B+\frac{\eta}{2}\cot 30^\circ\right. \tag{11}$$
$$\left.-\frac{\theta}{4}\tan B-\frac{\sigma}{2}\tan C\right]+\frac{\tau^2}{4}f\tan B-2\eta\sigma\tan B+\frac{\eta\sigma}{2}\frac{\cot C}{\cos^2 C}.$$

Dropping the second order terms from this equation we would have:

$$\theta=\eta(1-\cot 30^\circ\tan B)-\sigma(\cot C-\cot 30^\circ)\tan B.$$

Substituting this value for θ inside of the brackets of equation (11) and then simplifying, we obtain:

$$\theta=\eta(1-\cot 30^\circ\tan B)-\sigma(\cot C-\cot 30^\circ)\tan B-\frac{\eta^2}{4}\tan B\frac{n^2-1}{\cos^2 B}$$
$$+\frac{\tau^2}{4}f\tan B+\frac{\eta\sigma}{2}\frac{\tan B}{\cos^2 B}\left[(n^2-1)-(g^2-1)\frac{\cot 30^\circ}{\cot C}\right] \tag{12}$$
$$-\frac{\sigma^2}{4}\frac{\tan B}{\cos^2 B}\left[(n^2-1)+(g^2-1)-2(g^2-1)\frac{\cot 30^\circ}{\cot C}\right].$$

This equation can be written down for each of the three angles of the prism by giving subscripts 1, 2, 3 in turn to the quantities η, θ, σ, τ. Now if, as we are assuming, there is no pyramidal error,[5] the sum of the external angles of the prism $(A_1+A_2+A_3)$ is 180° and the sum of the angles of the liquid prism $(A_1-\sigma_1+A_2-\sigma_2+A_3-\sigma_3)$ is also 180°; hence, by (4),

$$\eta_1+\eta_2+\eta_3=0 \text{ and } \sigma_1+\sigma_2+\sigma_3=0. \tag{13}$$

Therefore the expression for $\bar{\theta}=\frac{1}{3}(\theta_1+\theta_2+\theta_3)$ is

$$\bar{\theta}=-\frac{1}{12}(\eta_1^2+\eta_2^2+\eta_3^2)\tan B\frac{n^2-1}{\cos^2 B}+\frac{1}{12}(\tau_1^2+\tau_2^2+\tau_3^2)f\tan B$$
$$+\frac{1}{6}(\eta_1\sigma_1+\eta_2\sigma_2+\eta_3\sigma_3)\frac{\tan B}{\cos^2 B}\left[(n^2-1)-(g^2-1)\frac{\cot 30^\circ}{\cot C}\right] \tag{14}$$
$$-\frac{1}{12}(\sigma_1^2+\sigma_2^2+\sigma_3^2)\frac{\tan B}{\cos^2 B}\left[(n^2-1)+(g^2-1)-2(g^2-1)\frac{\cot 30^\circ}{\cot C}\right]$$

Now by applying the difference of the last two relations of (4) three times, we obtain $60^\circ-\bar{\delta}=2B-\bar{\theta}$ and hence equation (9) gives

$$2\sin\tfrac{1}{2}(60^\circ-\bar{\delta})=2\sin\tfrac{1}{2}(2B-\bar{\theta})=\frac{\sin B}{\sin\frac{1}{2}(60^\circ)}[1-\tfrac{1}{2}\bar{\theta}\cot B-\tfrac{1}{8}\bar{\theta}^2]$$

[5] The pyramidal error influences the result less than the plate error; if appreciable, correction for it can be made in the usual manner.

which with equation (14) yields our final result

$$2\sin\tfrac{1}{2}(60^\circ-\bar{\delta}) = n\left\{1+\frac{1}{24}(\eta_1^2+\eta_2^2+\eta_3^2)\frac{n^2-1}{\cos^2 B}-\frac{1}{24}(\tau_1^2+\tau_2^2+\tau_3^2)f\right.$$

$$-\frac{1}{12}(\eta_1\sigma_1+\eta_2\sigma_2+\eta_3\sigma_3)\frac{1}{\cos^2 B}\left[(n^2-1)-(g^2-1)\frac{\cot 30^\circ}{\cot C}\right]$$

$$\left.+\frac{1}{24}(\sigma_1^2+\sigma_2^2+\sigma_3^2)\frac{1}{\cos^2 B}\left[(n^2-1)+(g^2-1)-2(g^2-1)\frac{\cot 30^\circ}{\cot C}\right]\right\} \quad (15)$$

In practice only the correction term depending on η^2 is of any importance; that term is the same as in the case of a solid prism and, as we have said, it is negligible if each of the three angles of the prism differs from 60° by less than ten minutes. Hence in all practical cases

$$n = 2\sin\tfrac{1}{2}(60^\circ-\bar{\delta}) \quad \text{where} \quad \bar{\delta} = \tfrac{1}{3}(\delta_1+\delta_2+\delta_3). \quad (16)$$

This provides a means for the accurate determination of n from measurements of *three similar* quantities, *viz.* the accurately measurable minimum deviations corresponding to the three angles of the prism.

3. USE OF THE INTERPOLATION FORMULA

In research work on the indices of refraction of liquids it is often desired not so much to obtain accurate values of the indices as it is to compare accurately the values of the indices of similar liquids, or of some one liquid at different temperatures, or of solutions of different concentrations. In such cases it is desirable to have a convenient method of calculating the value of N from the corresponding value of the minimum deviation D, when the values of n and n' and the corresponding values of δ and δ' measured by the same prism are known.

If a liquid is enclosed in a hollow prism, the covers of which are so nearly true plates that the plate correction σ may be disregarded, we obtain from

$$N = \frac{\sin\tfrac{1}{2}(A-D)}{\sin\tfrac{1}{2}A} \quad \text{and} \quad n = \frac{\sin\tfrac{1}{2}(A-\delta)}{\sin\tfrac{1}{2}A}$$

$$\cot\tfrac{1}{2}A = \frac{\cos\tfrac{1}{2}D-N}{\sin\tfrac{1}{2}D} = \frac{\cos\tfrac{1}{2}\delta-n}{\sin\tfrac{1}{2}\delta}$$

and further

$$\sin\tfrac{1}{2}(D-\delta) = n\sin\tfrac{1}{2}D - N\sin\tfrac{1}{2}\delta. \quad (17)$$

Similarly for the substances with indices N and n'

$$n'\sin\tfrac{1}{2}D - N\sin\tfrac{1}{2}\delta' = \sin\tfrac{1}{2}(D-\delta').$$

By multiplying the last two equations and rearranging the resulting equation, we obtain:

$$N \sin \tfrac{1}{2}(\delta' - \delta) = n \sin \tfrac{1}{2}(\delta' - D) + n' \sin \tfrac{1}{2}(D - \delta). \qquad (18)$$

Equation (18) would permit a simple interpolation (or extrapolation) for N, if D were measured and δ, δ', n, and n' were known.

If now the plate correction σ is to be taken into account, we start with an equation analogous to equation (3), namely

$$\frac{N}{1 - \frac{\sigma}{2}\left(\cot Z - \cot \frac{A}{2}\right)} = \frac{\sin \tfrac{1}{2}(A - D)}{\sin \tfrac{1}{2}A}$$

and similar expressions with n and n', and by repeating the processes that led to equations (17) and (18) we obtain:

$$\sin \tfrac{1}{2}(D - \delta) = \frac{n \sin \tfrac{1}{2}D}{1 - \frac{\sigma}{2}\left(\cot z - \cot \frac{A}{2}\right)} - \frac{N \sin \tfrac{1}{2}\delta}{1 - \frac{\sigma}{2}\left(\cot Z - \cot \frac{A}{2}\right)}$$

and

$$\frac{N \sin \tfrac{1}{2}(\delta' - \delta)}{1 - \frac{\sigma}{2}\left(\cot Z - \cot \frac{A}{2}\right)} = \frac{n \sin \tfrac{1}{2}(\delta' - D)}{1 - \frac{\sigma}{2}\left(\cot z - \cot \frac{A}{2}\right)} + \frac{n' \sin \tfrac{1}{2}(D - \delta)}{1 - \frac{\sigma}{2}\left(\cot z' - \cot \frac{A}{2}\right)}.$$

For sufficiently small values of σ the last two equations are equivalent to

$$N \sin \tfrac{1}{2}\delta = n \sin \tfrac{1}{2}D\left[1 - \frac{\sigma}{2}(\cot Z - \cot z)\right] - \sin \tfrac{1}{2}(D - \delta)\left[1 - \frac{\sigma}{2}\left(\cot Z - \cot \frac{A}{2}\right)\right] \qquad (19)$$

and

$$N \sin \tfrac{1}{2}(\delta' - \delta) = n \sin \tfrac{1}{2}(\delta' - D)\left[1 - \frac{\sigma}{2}(\cot Z - \cot z)\right] + n' \sin \tfrac{1}{2}(D - \delta)\left[1 - \frac{\sigma}{2}(\cot Z - \cot z')\right]. \qquad (20)$$

If equation (20) is examined carefully, it will be found that the two terms $-n \sin 1/2(\delta' - D)\sigma/2(\cot Z - \cot z)$ and $-n' \sin 1/2(D-\delta)\ \sigma/2$ $(\cot Z - \cot z')$ by which equation (20) differs from equation (18) are always of *opposite* sign and very nearly equal. This is true whether N lies between n and n' or not. Therefore the value of N obtained from equation (20) is so nearly equal to that obtained from equation (18), that equation (18) may be taken as the *exact* interpolation formula. The fact that the simple formula (18) is exact, even where the plate correction is appreciable, is the advantage obtained by using *two* known substances. If, on the other hand, *one* known substance only were to be used, the calculation should be made by equation (19). The examination of this equation shows that the terms by which it differs from equation (17) do *not*, as a rule, cancel in the manner that the terms of equation (20) do, and accordingly the simple formula (17) may *not* be used unless either σ is so small or N so near g that the plate correction becomes imperceptible.

To illustrate and verify these statements, values of δ, D, and δ' corresponding to the order of magnitude $n = 1.2$ and $n' = 1.3$ were assumed as measured to seconds and the exact values of n and n' were obtained by means of equation (3) in which g was taken as 1.70, σ as $10''$, and A as $60°$ exact.[6] The values of N were then found from assumed values of D by means of equations (18) and (20), and also from equation (3). All three values agreed as closely as the calculations could be made with seven place tables, *i.e.* the values never differed by more than one unit of the sixth decimal and usually not even as much. It is evident that if similar calculations were made with values of g lying closer to those of n than those that we assumed, the effect of the plate correction would be less marked and our conclusions would be *a fortiori* valid.

Similar calculations were made with the same sample values to test equations (17) and (19). According to expectation equation (19) yields the correct value of N, but the value of N obtained from equation (17) differs from all other values by at least as much as one unit of the *fifth* decimal.

SUMMARY

The index of refraction of a liquid enclosed in a hollow prism may be calculated from

$$n = 2 \sin \tfrac{1}{2}(60° - \bar{\delta}),$$

[6] It should, however, be emphasized here that the conclusions of this section are valid whether A is $60°$ or not. The value of A does not affect the answers provided it is the same in all equations.

$\bar{\delta}$ being the average of the three minimum deviations. It is not necessary that the covers be true plates, but the three angles of the prism should not differ much from 60°, preferably by no more than 10′.

If measurements are made with the same prism containing different liquids, the interpolation formula

$$N \sin \tfrac{1}{2}(\delta' - \delta) = n \sin \tfrac{1}{2}(\delta' - D) + n' \sin \tfrac{1}{2}(D - \delta)$$

is exact, even when the covers depart perceptibly from true plates. The prism angle A may have any value.

If one known substance only is available, N may be obtained accurately from

$$N \sin \tfrac{1}{2}\delta = n \sin \tfrac{1}{2}D\left[1 - \frac{\sigma}{2}(\cot Z - \cot z)\right] - \sin \tfrac{1}{2}(D - \delta)\left[1 - \frac{\sigma}{2}\left(\cot Z - \cot \frac{A}{2}\right)\right].$$

THE ZINC SULPHIDE METHOD OF MEASURING ULTRAVIOLET RADIATION AND THE RESULTS OF THREE YEARS' OBSERVATIONS ON BALTIMORE SUNSHINE

By Janet H. Clark

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The results of the measurements of solar ultraviolet radiation in Baltimore from October, 1927, to May, 1930, have already been published.[1] However, as the annual variation in ultraviolet radiation is of interest to physicists as well as to physiologists, the main results obtained, including measurements up to November, 1930, are summarized briefly here.

Method

A few years ago the author published a chemical method for measuring the intensity of ultraviolet radiation by means of the rate of darkening of a light sensitive paint, containing zinc sulphide, known as lithopone.[2] The method is similar to the measurement of intensities with a photographic plate except that lithopone is only sensitive to radiation shorter than 350 mμ. The sensitivity curve of lithopone has been determined by Pfund and Brickwedde.[3] The method proved useful in estimating the intensity of ultraviolet radiation from quartz mercury arcs in which the wave lengths effective in darkening the lithopone are λ313mμ and shorter, and are the same wave lengths that are effective in therapy.

As the light sensitivity of lithopone is extremely variable, the method has been improved by substituting C. P. zinc sulphide. Chemically pure zinc sulphide, when prepared by different methods, differs in its sensitivity to ultraviolet radiation. A standard grade known as "ZnS ignited" has been developed by the J. T. Baker Chemical Company for this work and found very satisfactory. A spatula full of this zinc sulphide powder is put in small mortar, moistened with a few drops of a saturated solution of lead acetate, and ground to a smooth, soft paste. This paste is placed on a piece of glass and pressed flat under a piece of transparent quartz. It is necessary to expose under quartz as drying of the paste lessens its sensitivity, and as the paste darkens on exposure to the air, fresh material should be used for each observation. The glass and

quartz plates, with the zinc sulphide paste between them, are held together with elastic bands and the paste is exposed normally to the radiation through the quartz (see Fig. 1).

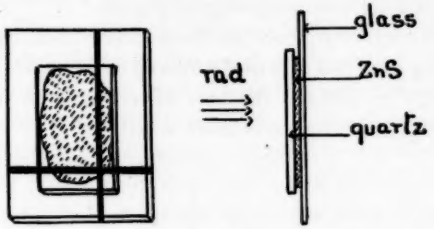

FIG. 1. *Method of exposing ZnS paste.*

Before exposure the ZnS paste has a reflection factor of 70 percent. It darkens rapidly on exposure to ultraviolet radiation and the darkening is irreversible. After exposure of 1, 2, 3, etc. minutes the reflection factor of the darkened ZnS is determined by means of a Macbeth Illuminometer. From the curve of darkening thus obtained (see Fig. 2), the

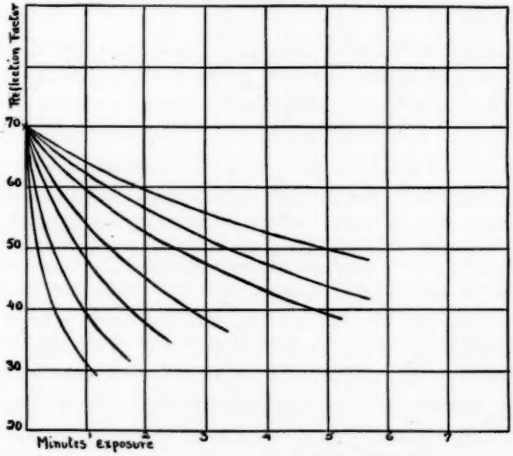

FIG. 2. *Darkening curves of ZnS "ignited" lot #S102428.*

exposure time is found which is necessary to produce a darkening to a 50 percent reflection factor. This is the time necessary to give one ZnS unit of ultraviolet radiation and the reciprocal of the time is the intensity in terms of ZnS units. In practice a set of typical darkening curves,

such as those given in Fig. 2, will enable the observer to determine the time to give one unit from one observation only. If no illuminometer is available the reflection factor can be determined reasonably well by comparison with a set of standard gray papers.*

The sensitivity curve of ZnS has been determined by comparing its rate of darkening with the rate of darkening of lithopone under different filters. From 350 to 290 mμ the two have the same sensitivity. For shorter wave lengths lithopone is more sensitive.

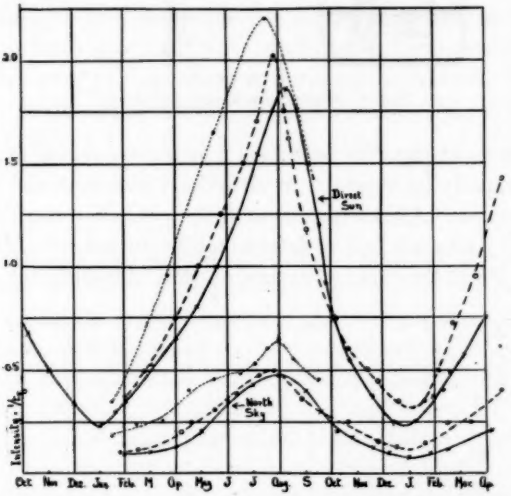

FIG. 3. *Annual variation in solar ultraviolet radiation (290–350 mμ) at noon on clear days in Baltimore. abscissae: Time in months. ordinates: Intensity of ultraviolet radiation (reciprocal of time required to give one ZnS unit).* ×——× *direct sun and north sky readings October 1927–April 1929.* ○ — — ○ *direct sun and north sky readings February 1929–April 1930.* ●-------● *direct sun and north sky readings January 1930–October 1930.*

If the zinc sulphide method is used to measure the intensity of a quartz mercury arc only wave lengths 313 mμ and shorter are effective. If it is used with the mercury arc through a Corning G 986 A filter practically all the darkening is produced by wave lengths 313-289 mμ. The energy necessary to give one ZnS unit in this region has been measured by means of a calibrated galvanometer and thermopile and was found to be equivalent to 720,000 ergs per sq cm.

When the method is applied to the measurement of solar radiation it is less satisfactory because the darkening is then due to a band from

* The ZnS *YR* chart from the Munsell Color Co., Baltimore, Md.

290-350mμ, with the maximum effect at 320mμ. From October, 1927, to October, 1930 measurements were made in Baltimore on clear days at noon, both of direct sun and north skylight. Zinc sulphide was exposed under quartz to direct sun at the bottom of a blackened box, so as to cut off the effect of skylight, and also on a north window sill to the radiation of one half the sky. The time to give 1 ZnS unit was determined and the results obtained are plotted in Fig. 3. In the first year the readings showed a maximum intensity early in August and a minimum intensity about the first of January, the ratio of the two being 8:1. The intensity in terms of zinc sulphide units (the reciprocal of the time necessary to give one unit) varied from 1.8 units in August to 0.23 units in December. During the second year the intensity rose more rapidly in the spring and reached a maximum towards the end of July. The ratio of maximum: minimum was 8.6:1 and the intensity varied from 2.07 units in July to 0.24 units in December. During the third year the winter minimum was not so low (0.32 units in December). The intensity increased rapidly in the spring and reached a maximum of 2.2 units late in July. The ratio of maximum: minimum was 7:1.

The method as described measures the intensity of a band from 290 to 350mμ, only part of which lies in the therapeutic region. As it was desirable to measure the energy of the antirachitic region (290-315mμ) the method was subsequently modified and from October, 1928, to October, 1930, simultaneous readings were made of the rate of darkening of zinc sulphide under glass and quartz, the glass being a white microscope slide 1 mm thick. Since 1/t quartz is the intensity of λ 290 – 350mμ and 1/t glass is the intensity of λ 315 – 350 mμ, the intensity of the therapeutic region λ 290 – 315 mμ is given by 1/t quartz – 1/t glass. As the value of the zinc sulphide unit has been determined in this region the results can be expressed in ergs per sq cm per sec and are given by $(1/tq - 1/tg) \times 720{,}000/60$. The results for direct sun and north sky are given in Fig. 4. It was found that the antirachitic radiation in Baltimore varied in 1928-29 from 400 ergs per sq cm per sec in December to 5800 ergs per sq cm per sec in July, which gives a ratio of 1:14.5. The antirachitic radiation in north skylight varied from 300 ergs per sq cm per sec in December to 2100 ergs per sq cm per sec in July, of the same year, giving a ratio of 1:7. In 1929-30 the antirachitic radiation varied from 600 ergs per sq cm per sec in December to 6200 ergs per sq cm per sec in July, which gave a ratio of 1:10. The shape of the curves during these two years is very similar.

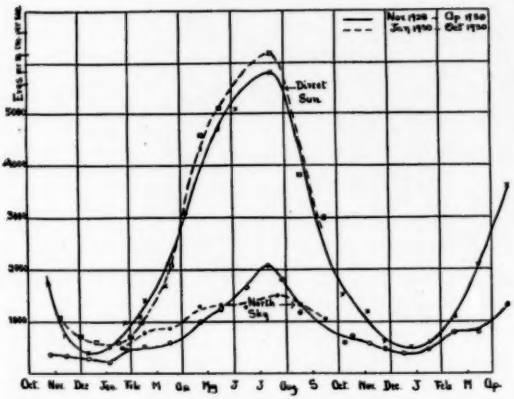

FIG. 4. *Annual variation in the antirachitic radiation (290–315 mμ) from direct sun and north sky in Baltimore on clear days at noon. abscissae: time in months. ordinates: intensity of ultraviolet radiation in ergs per sq cm per sec.*

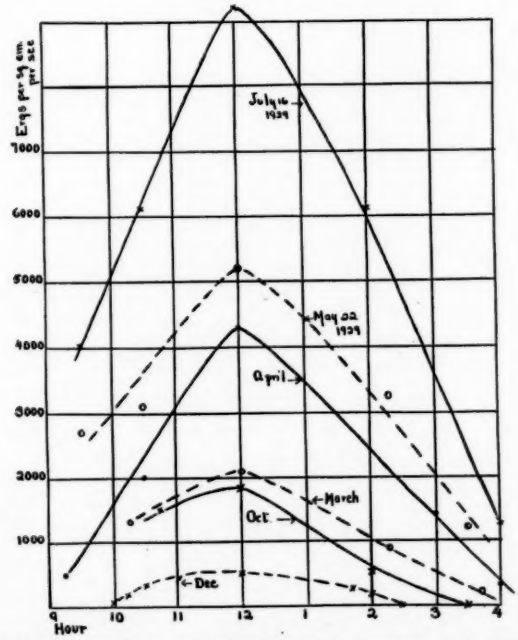

FIG. 5. *Variation in antirachitic radiation (290–315 mμ) throughout the day in different months (direct sun).*

On certain very clear days readings were made throughout the day. In Fig. 5 the variation throughout the day is given for the antirachitic region 290 – 315mμ. The only day on which readings were made in July was a day giving exceptionally high noon readings and is consequently above the average for the month. Except for May 22 and July 18 the curves represent average results for a number of days. The antirachitic radiation has approximately the same intensity in March and October.

North sky readings were made both on clear and on cloudy days and it was found that unless the clouds were very dense they had little effect on the antirachitic radiation which was frequently more intense on cloudy days. (see Table 1.)

TABLE 1. *Antirachitic radiation (λ290–315 mμ) from north sky.*

Date	Weather	Intensity ergs cm^{-2} sec^{-1}
May 10	Almost no clouds, visibility good	1032
16	Sky almost covered with clouds	1800
21	Sky covered with light clouds, visibility good	1020
June		
average	Clear days	1650
average	Cloudy days	1960
July		
average	Clear days	2040
average	Cloudy days	1860

COMPARISON WITH OTHER RESULTS

Since readings made in the region 290-315 mμ can be expressed in ergs per sq cm per sec, they can be compared with readings given by Coblentz[4] for the ultraviolet radiation shut out by common window glass, in Washington, which are given in gr cal per sq cm per min. When both sets of readings are expressed in ergs per sq cm per min (see Table 2), they show very good agreement.

TABLE 2. *Ultraviolet radiation shut out by window glass.*

Month	Ergs per sq cm per min	
	Coblentz	Clark
June	280000	302520
March	160000	130800
January	40000	42000

Readings were also made during 1928 and 1929 by the ZnS method at Boulder, Colorado. The results have been reported by Earp[5] and the difference between the climates of Boulder and Baltimore in respect to ultraviolet radiation, is discussed in his papers.

EFFECT OF OZONE IN THE UPPER ATMOSPHERE

In 1928 the maximum intensity of solar ultraviolet radiation came early in August, and in 1929 and 1930 late in July. Dorno with the cadmium cell found the maximum in July at Davos and Hill with the methylene blue method found his highest readings either in July or August. In the previous paper[1] it was suggested that the shift of intensity maximum from June to July or August may be accounted for by the annual variation in the ozone in the upper atmosphere. Dobson's[6] and Buisson's[7] measurements show that the amount of ozone varies through the year with a maximum in the spring and a minimum in the autumn. This would tend to displace the ultraviolet maximum to a date later than June 22, when the solar altitude is greatest.

TABLE 3.

	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.
Dobson	d=cm of pure ozone at normal temperature and pressure.								
1925 Oxford	.340	.304	.340	.321	.296	.289	.273	.266	.239
1926 Oxford	.278	.290	.299	.313	.301	.266	.258	.228	.232
Arosa						.253	.244	.230	.224
1927 Oxford		.340	.320	.319	.312	.288	.275	.254	.237
Arosa	.293	.302	.301	.306	.282	.272	.249	.246	.228
1928 Oxford		.296	.323	.317	.294	.259		.243	.238
Arosa	.261	.279	.290	.301	.266	.248	.238	.232	.216
Buisson									
1927 Marseilles	.325	.352	.361	.353	.316	.292	.276	.276	.268
1928 Marseilles	.316	.344	.339	.328	.293	.281	.261	.265	.261
	Effective depth of ozone $=d_1=d/\cos\theta_1$								
1925 Oxford	.551	.410	.392	.342	.308	.303	.300	.333	.344
1926 Oxford	.451	.390	.345	.333	.313	.280	.283	.286	.334
Arosa						.266	.267	.288	.323
1927 Oxford		.457	.369	.339	.324	.303	.301	.318	.341
Arosa	.475	.407	.347	.326	.293	.286	.273	.308	.329
1928 Oxford		.399	.373	.337	.306	.272		.304	.343
Arosa	.424	.375	.334	.320	.277	.261	.261	.291	.311
1927 Marseilles	.527	.473	.416	.375	.329	.307	.302	.346	.332
1928 Marseilles	.467	.463	.392	.360	.305	.295	.286	.332	.375

The effective depth of ozone equals $d_1 = d/\cos\theta_1$, where d is the actual thickness of the ozone layer and θ_1, is the zenith distance of the sun. From Dobson's value for d measured from 1925-1928 at Oxford and Arosa, and from Buisson's value for d measured at Marseilles in 1927 and 1928, the effective depth of this layer was calculated for noon sun. The results given in Table 3 show that the effective depth of this layer is least in July or August. The minimum value occurred early in August in 1928, which agrees with the ultraviolet radiation maximum.

CONCLUSION

Measurements of the solar ultraviolet radiation have been made in Baltimore over a period of three years by the zinc sulphide method. The annual variation of the noon readings, both for direct sun and for north skylight is given over this period of time for the region 290-350mμ and the values of the noon readings for the therapeutic region 290-315mμ are given from October, 1928 to October, 1930. The therapeutic radiation in direct sun at noon, as measured by this method, varies from about 500 ergs, per sq cm per sec in December to about 6000 ergs per sq cm per sec in July, which gives a ratio of 1:12. The therapeutic radiation in north skylight was found to vary from 300 ergs per sq cm per sec in December to 2100 ergs per sq cm per sec in July, giving a ratio of 1:7.

REFERENCES

1. Clark, J. H.: Am. J. Hygiene, *9*, 646; 1929. Am. J. Hygiene, *12*, 690; 1930.
2. Clark, J. H.: Am. J. Physiology, *69*, 200; 1924.
3. Brickwedde, F. G.: J.O.S.A., *14*, 312; 1927.
4. Coblentz, W. W. and R. Stair: Bureau of Standards, J. Research, *3*, 629; 1929. Research paper No. 113.
5. Earp, J. R.: Am. J. Hygiene, 9, 663; 1929. Am. J. Hygiene, *12*, 696; 1930.
6. Dobson, G. M. B.: Proc. Roy. Soc. A., *110*, 660; 1926. Proc. Roy. Soc. A., *114*, 521; 1927. Proc. Roy. Soc. A., *122*, 456; 1929.
7. Buisson, H.: Comptes Rend. Acad. Sci. *186*, 1229; 1928. Comptes Rend. Acad. Sci. *188*, 647; 1929.

APPLICATION OF THE SLITLESS SPECTROGRAPH TO THE MEASUREMENT OF THE DOPPLER SHIFT: SUPPLEMENTARY NOTE

By Noel Deisch
[908 G. St., Washington, D. C. Received Jan. 9, 1931]

Since the publication of the article on the above subject in the December, 1930, issue of this *Journal*, Dr. Otto Struve in a private communication has called the writer's attention to earlier work on the problem of determining radial velocities from spectrograms taken with objective prisms, giving references which are indicated below.

A differential method broadly resembling that under consideration was suggested in 1893 by Edwin B. Frost[1] in a letter to E. C. Pickering. According to this proposal, spectrograms as ordinarily obtained were to be used, the separation of lines located near opposite terminators of any spectrum included in the field being compared with the separation of similar lines in a previously studied spectrum also included in the field, measurement being made of the distance between similar lines in the two spectra to be compared. A. Orbinsky[2] independently suggested an analogous method, the two spectra in this case however being brought into close relation by successive exposures.

The device of using spectra in reversed relation to double the differential displacement was broached by E. C. Pickering[3] in 1896. According to the earlier form of his proposal, photographs are taken on separate plates, one exposure being made through the glass (i.e., with the emulsion side of the plate away from the lens) and with the prism reversed. The plates are then superposed with the spectra contiguous, one plate being moved over the other for measurement by means of a micrometer screw. By this means corresponding lines in the two images of each star in turn are made to coincide, and the differences in the readings gives double the displacement of the line. A preliminary test was made with a small instrument in 1896, and, though the shift was shown, the practical working of the method evidently was not attempted.

In a further note on the above subject written some ten years later, Prof. Pickering[4] states that "It is not necessary or advisable to use two plates." He gives a photograph showing a trial spectrogram secured

[1] Astrophysical J., *2*, 1895; 235.
[2] Astronomische Nachrichten, *138*, No. 3289; 1895.
[3] Harvard College Observatory Circular, No. 13; 1896.
[4] Astrophysical J., *23*, 1906; 255.

with the 11 inch Draper telescope using a method substantially identical with that illustrated in Fig. 1 of the *Journal* article above referred to. In the specimen photograph, however, the two sets of spectra are separated by some distance, no provision having apparently been made in the experimental mounting of the telescope whereby the spectra might be brought into actual juxtaposition. Prof. Pickering estimates that, assuming an accuracy in the measurements of $\pm 0.''1$, the corresponding probable error would be ± 3.5 km/sec., but no results of measurements are given. In speaking of the general method he says: "The principal sources of error, such as those due to the distortion of the lens and changes of temperature, are radial, and may be determined, using both co-ordinates in the lines of all the spectra. Changes in the differential refraction may be reduced, if desired, by turning the prism so that the spectra shall be horizontal, instead of vertical."

Dr. Struve in his letter to the writer, above referred to, states that the practical application of the method has not met with great success, considerable difficulties having been experienced in the elimination of distortions due to the optical system. In this connection it may be noted that, in the case where a given line in the two spectra is brought into practical coincidence on the plate, as shown at *B* and *C* in Fig. 2 of the *Journal* article, the effects of distortion for that line should be virtually eliminated, due to the fact that the bundle of rays representing the image of this line would follow equal zonal paths through the instrument in each exposure: the distortions in the two cases would hence be equal and should cancel out. It would be futile to attempt the practical application of such a method, however, with any but a carefully designed special mounting.

A STUDY OF THE RELATION OF THE INTENSITY OF FLUORESCENCE TO THE CONCENTRATION IN THE CASE OF SOLID SOLUTIONS

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Received January 30, 1931]

Introduction

A considerable amount of work, both experimental and theoretical, has been done by various authors,[1,2,3,4] and[5], to find the law of variation of fluorescence with concentration. While a great deal of this work has been confined to liquid solutions, some investigations have included both liquid and solid solutions. Except for small values of the concentration the experimental results of different observers are in good agreement and those who have discussed the matter from the theoretical side agree in the conclusion, verified by experiment, that the specific fluorescence should diminish exponentially as the concentration increases.

There is considerable uncertainty however as to the cause of the decrease and especially as to the extent to which diffusion of the active material and collisions of the second kind are to be regarded as important factors. The fact that the specific fluorescence diminishes in solid solutions as well as in liquids as the concentration increases, and apparently by the same law, shows that diffusion cannot be an essential factor. But on the other hand the fact that the decrease is more rapid in liquid solutions than in solid solutions suggests that diffusion cannot be entirely ignored. Determinations of the law of variation with concentration for the same substance both in solid and liquid solutions have been made in only a few cases. The measurements described in the present paper were made with the object of increasing the amount of experimental data of this nature.

Experimental Procedure

Measurements of fluorescence as a function of concentration have been made for solutions of rhodamin-B in collodion both in the liquid form and after evaporation to form solid films.

[1] Bruninghaus, M. L., Comptes Rendus, *149*, 1375; 1909. Bruninghaus, M. L., Comptes Rendus, *149*, 1124; 1909.

[2] Perrin, J. Comptes Rendus, *177*, 469; 1923.

[3] Perrin, F. Comptes Rendus, *178*, 1978, 1924. Perrin, F. Comptes Rendus, *178*, 2252; 1924.

[4] Wawilow, S. J. ZS. f. Phys. *31*, 750; 1925.

[5] Merritt, E., J.O.S.A. *12*, 613; 1926.

Preparation; The solutions used in this work were made from a stock solution of high concentration. The stock solution was prepared by dissolving a known weight of rhodamin-B (Heller and Merz) in a known weight of absolute ethyl alcohol. In preparation of the solutions for final use some of the stock solution was mixed with absolute ethyl alcohol and collodion. The ratio by weight of the alcohol to the collodion in all the solutions was made four to one. Clear collodion was used in all cases. The stock solution and the solutions of various concentrations were kept in bottles which were tightly closed to prevent evaporation and were kept in the dark.

The cell used for the liquid solutions consisted of two glass plates separated by a glass ring 2.08 mm thick. The faces of the ring and the plates were optically plane. A small gap in the ring about 2 mm in width served as an opening for filling the cell. To reduce the thickness of the liquid layer still further disks of plane parallel glass were placed in the cell. The one generally used being 1.95 mm thick. The use of a thin layer of the solution prevented to a large extent the absorption of the fluorescent light, especially on the long wave-length side of the fluorescent band where all the intensity measurements were made.

In preparing the solid solutions glass plates were carefully cleaned and placed on a glass shelf in an oven. This shelf was leveled so that with a glass plate in position, the liquid solution would spread out forming a film approximately uniform in thickness. The oven was blackened on the inside and closed with a black cloth, thus keeping the specimens in the dark while drying. Each glass plate was labeled and placed in an envelope when removed from the oven. All of the specimens were kept in the dark except when used in tests.

MEASUREMENTS OF FLUORESCENCE

The experimental arrangement is shown in Fig. 1. The lens system l was used to focus the filament of a 500-watt tungsten lamp L on the slit B of a Bausch and Lomb monochromatic illuminator. By means of a wave-length drum the prism P_3 was adjusted to give the part of the spectrum desired at slit A. The band of light from A could either be used to excite fluorescence in the specimen placed at M or could be reflected by a mirror into the slit S_1 of the collimator C_1 of a Lummer-Brodhun spectrophotometer. The lamp L was also used as a comparison source. Stray light from L was avoided by screens forming the narrow box D which was light tight except for the slit at each end.

Several pieces of ground glass, G, placed in front of the slit S_2 of collimator C_2 served to reduce the intensity to a suitable extent. The Lummer-Brodhun cube P_2, the dispersing prism P_1 and the telescope T made up the remainder of the spectrophotometer. The slits of the monochromator and the spectrophotometer were adjustable and their widths could be obtained by reading the drum attached to each slit. The observations were made in a dark room so that light from any outside source was eliminated.

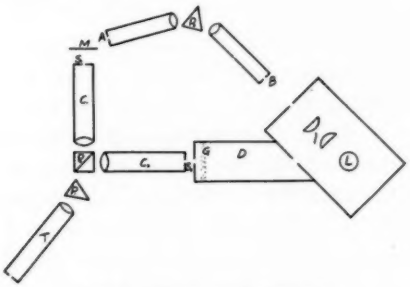

FIG. 1. *Diagram of apparatus.*

For measurements of the intensity of the fluorescent light the width of S_1 was constant and a balance was obtained by adjusting S_2. The same method was used in making measurements on the transmission of the specimens.

In measurements on both liquid and solid solutions where a broad exciting band was used to excite fluorescence, the specimen at M was set so that the plane made an angle of 11 degrees with the exciting band of light from slit A. This position of the specimen caused the band of exciting light to be spread over a fairly large area of the fluorescence material. The path of the exciting light for this position was more than five times the actual thickness of the solid film or liquid solution while the fluorescent light, even when produced at the surface of the active material away from S_1, had to tranverse a distance in the solution not greater than its thickness. Scattered light from any part of the exciting band for this arrangement was very small, and for the region in which the measurements of fluorescence were made, could not be detected. In the greater part of this work readings on the intensity of fluorescence were taken only on the extreme edge of the long wave-length side of the fluorescent band, since absorption of the fluorescent light by the solution could be neglected in this part of the spectrum.

MERCURY ARC AS EXCITING SOURCE

In some experiments the green line of a Gallois quartz mercury arc which falls near the middle of the absorption band of rhodamin was used as the exciting source, with the same tungsten lamp previously used as the comparison source.

The mercury arc with its fixed resistance was connected in parallel with the tungsten lamp L in Fig. 1. A choke coil and a variable resistance were placed in the circuit to keep the current through the arc and lamp constant. A 110 dc generator provided with a voltage regulator proved to be satisfactory as a current source. A voltmeter connected across the two lamps in parallel and an ammeter in the mercury arc branch were observed between readings.

The mercury arc was enclosed in a tin box provided with a hole one inch in diameter opposite the most intense part of the arc. A short focal length lens was used to make the rays of light sufficiently convergent so that the desired portion of the specimen was covered. The beam of light passed through a heat absorbing glass and a green filter before reaching the specimen. The green filter transmitted the green line of the mercury but the yellow lines were almost completely absorbed. A frame work covered with black cloth prevented stray light from reaching any part of the apparatus. A small electric fan controlled the temperature so that the arc could be run continuously. The lamps were allowed to run for half an hour before taking a set of readings in order to insure constant illumination.

In the measurements on fluorescence the specimens were placed in the same position M, Fig. 1 as before but the angle between the incident beam and the plane of the specimen was 20 degrees, since this position was found by trial to be best suited for the observations in this part of the work.

TRANSMISSION MEASUREMENTS

In measurements on transmission a mirror was placed at M, Fig. 1 but set to reflect the light from slit A into S_1. A glass plate on which a thin film of the alcohol—collodion solution had been dried was placed in front of S_1 and S_2 was adjusted until a balance was obtained. Readings were taken at equally spaced intervals covering the exciting band. A similar glass plate, with a thin film of the collodion-rhodamin solution dried on it, was inserted in the same position as the first plate and readings were taken over the same range as before. Transmission data for the liquid solutions were obtained in the same way with the exception

that the solution was placed in the thin cell used for fluorescence measurements. All readings were corrected for slit width according to the work of Nichols and Merritt[6].

The intensity of the transmitted light measured in this part of the work and for the previous case where a broad band was used, was for perpendicular incidence. In order to use this data in correcting the fluorescence measurements, the intensity of the light transmitted through the liquid and solid solutions was calculated for oblique incidence as used in all fluorescence measurements. These computations could be readily made since the intensities of the incident and emergent beans for perpendicular incidence were known and the angle between the plane of the specimen and the incident ray was measured in every case.

SOLID SOLUTIONS

TUNGSTEN LAMP AS EXCITING AND COMPARISON SOURCE

When a broad band is used to produce fluorescence, it is necessary to know the energy distribution for this band in order to determine the amount of energy absorbed and available to produce fluorescence. The energy distribution in the spectrum of lamp L, Fig. 1 was determined by comparison with a 40-watt tungsten lamp which had been calibrated by the Bureau of Standards and found to give the same radiation as a black body at 2300 degrees K when the potential difference between the terminals was maintained at 97.3 volts, which was the voltage used in this test. The energy distribution for this lamp was computed by the aid of Planck's equation, the constants used being those given by Coblenz.[7] The intensity distribution throughout the band having been determined by comparison with lamp L, the band was reduced to an energy basis by multiplying each intensity ordinate of the band by the corresponding one of the energy distribution curve for lamp L. The energy distribution obtained in this way is shown by E, Fig. 2. Likewise the transmission curves for each concentration were reduced to an energy basis and these curves 2, 3, 4, 5, 6, and 7 are also shown in Fig. 2.

The area under the energy distribution curve of the exciting band E Fig. 2 represents the energy available. The difference between this area and that under the transmission curve for a given concentration gives the energy actually absorbed and is therefore proportional to the fluorescence produced. By multiplying the intensity of fluorescence observed by the ratio of the total energy to the absorbed energy, the intensity of

[6] Nichols and Merritt, Studies in Luminescence Carnegie Inst. Pub. No. 152, p. 168.
[7] Coblenz, W. W., Bulletin of Bureau of Standards *15*, 618; 1918–1920.

fluorescence is obtained corresponding to an absorption of all the energy. The observed fluorescence was corrected in this way for each solution.

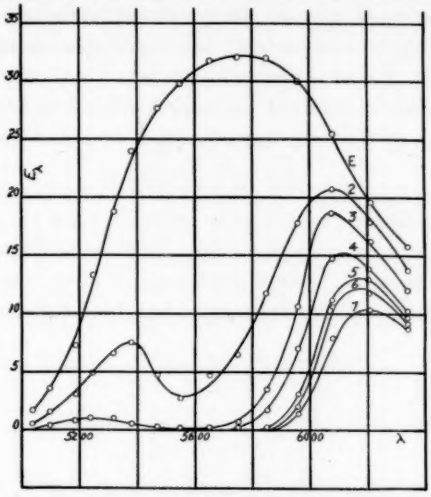

FIG. 2. *Transmission curves for solid solutions and solvent. Energy plotted against wave length. Concentrations (2) 2.98×10⁻⁶, (3) 10.2×10⁻⁶, (4) 16.8×10⁻⁶, (5) 21.7×10⁻⁶, (6) 25.9 ×10⁻⁶, (7) 38.2×10⁻⁶ gm mol/cc.*

The points marked by crosses in Fig. 3 were computed upon the assumption that the exciting power is independent of the wave length as

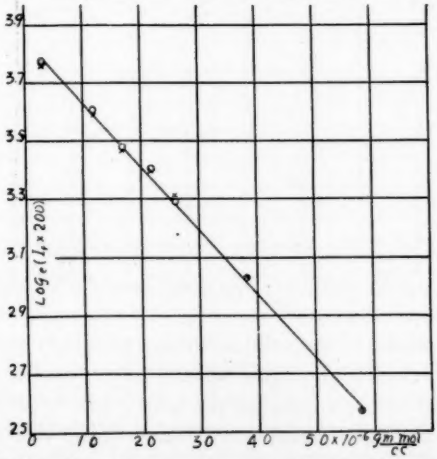

FIG. 3. *Relation of the intensity of fluorescence to concentration for solid solutions. Specific exciting power proportional to wave length is shown by circles; independent of wave length by crosses.*

found by Wawilow.[8] The circles, Fig. 3, are computed on the assumption that the exciting power is proportional to the wave length, this law being in approximate agreement with the results of Nichols and Merritt.[9] It will be seen for the narrow exciting band used there is very little difference between the results obtained. No corrections were made for absorption of the fluorescent light since all the observations were made on the extreme edge of the long wave-length side of the fluorescent band.

MERCURY ARC EXCITATION

Since there are fewer corrections to be made when the exciting source is monochromatic, greater weight should be given to the results obtained in this part of the work. The fluorescence readings were taken on the long wave-length side of the fluorescent band for the reason previously stated.

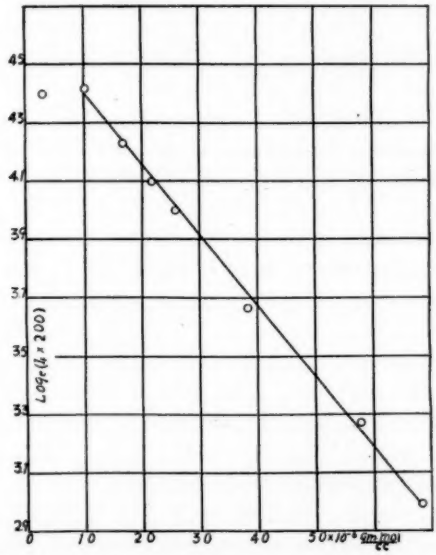

FIG. 4. *Fluorescence produced by green line of mercury arc in solid solutions.*

The concentrations of the solid solutions were obtained by multiplying the concentrations of the liquid solutions by the ratio of a known volume of the liquid to the volume of the solid solution resulting from the evaporation of the liquid solution. To determine this ratio, a known

[8] Vavilov (Wawilow) Phil. Mag. *43*, 307; 1922.
[9] Nichols and Merritt, Phys. Rev. *31*, 381; 1910.

volume of the liquid solution, contained in a glass box of known weight and volume, was allowed to evaporate under the same conditions as used in the preparation of the solid films. By weighing the box with the solid solution, then filling with distilled water and reweighing the volume of the solid was found. These measurements gave a value of 47.8 for the ratio of the volume of the liquid solution to the volume of the solid solution.

The results shown in Fig. 4 are very similar to those obtained for the same solutions with the broad exciting band. Only one point seems too far from the curve to be accounted for by errors made in observations. Except for this one point the intensity of fluorescence is seen to decrease exponentially with an increase in concentration over a considerable range of concentration. By applying Perrin's equation.

$$I = I_0 e^{-kz}$$

to the data for the solid solutions, the value of k is found to be 2.48×10^4.

LIQUID SOLUTIONS

The liquid solutions from which the solid solutions were made, were tested in the same manner as the solid solutions. One set of measure-

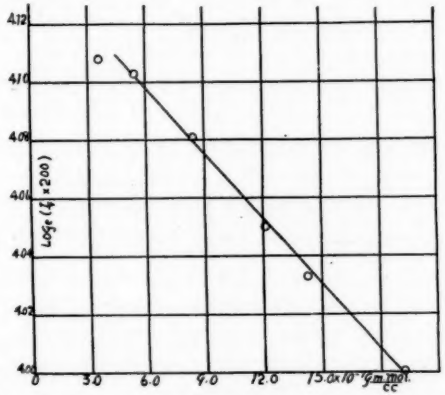

FIG. 5. *Fluorescence produced in liquid solutions by green line of mercury arc.*

ments with the green line of the mercury arc as the exciting source is represented in Fig. 5. The value of k obtained for the liquid solutions is 7.27×10^4 which is about 2.9 times larger than for the solid solutions.

On account of the small intensity of the fluorescence reliable measurements with less concentrated solutions could not be made.

SUMMARY

(1) The results, both for liquid and solid solutions, show that the intensity of fluorescence increases exponentially with a decrease in concentration through a wide range. These results are in agreement with the experimental work of others and also with the different theories; for all of the theories proposed call for an exponential decrease in intensity with an increase in concentration for the range of concentrations covered in these experiments.

(2) Wawilow[4] found that the fluorescence power reached a constant value for liquid solutions of small concentrations. The most reliable test, Fig. 4, for the solid solutions may indicate a change similar to that observed by Wawilow for the change noted near the lowest concentration seems too large to be accounted for by observational errors. There is some evidence of a similar change in case of liquid solutions Fig. 5. However, sufficient data were not taken in the region of small concentrations to determine the law of variation in this region.

(3) The value of k, (i.e., the multiplier of the concentration in the exponential expression), for liquid solutions was found to be about 2.9 times larger than for solid solutions. According to this result, diffusion of the active material in liquid solutions is an important factor as the work of J. Perrin[2] and F. Perrin[3] has shown. If it is assumed that the fluorescence power of a molecule of the active material is destroyed when nearer than a critical distance p of another active molecule, then p can be expressed in terms of k. By using the values of k determined for liquid and solid solutions, the calculation of $p(2.13\times10^{-7})$ for liquid solutions is found to be larger than $p(1.82\times10^{-7})$ for solid solutions.

The writer wishes to express his indebtedness to Professor Ernest Merritt at whose suggestion and under whose direction this work was done.

THE ABSORPTION OF LIQUID AND SOLID SOLUTIONS OF RHODAMIN-B IN A MIXTURE OF ALCOHOL AND COLLODION

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In studying the relation of the intensity of fluorescence to the concentration in the case of liquid and solid solutions of rhodamin-B in alcohol mixed with collodion, it was necessary to measure the absorption of these solutions. The transmission data obtained may be of interest since these results may be compared with those of other workers in this field.

The work of Holmes[1] and Speas[2] has shown that two absorption bands are present in aqueous solutions of rhodamin-B. Speas also observed two bands in alcoholic solutions of rhodamin-B. These bands were found to shift towards longer wave lengths with an increase in concentration.

Results

The experimental procedure has been described in a previous report and need not be repeated here. While no effort was made to control the temperature it was at least approximately constant, since all readings were taken at room temperature. The absorption curves may be assumed to represent changes due to concentration and not to temperature.

The liquid solutions contained four parts of alcohol by weight to one of collodion. The inside thickness of the cell used in testing the liquid solutions was .013 cm. The solid films were made by evaporating the liquid solutions on clean glass plates. No attempt was made to determine the thickness of each film tested, but from measurements on some of the films, the thickness was estimated to be less than .001 cm. However, the ordinates for all the curves represent the values of kd obtained from the equation

$$I = I_0 e^{-kdc}$$

where I_0 and I represent the intensities of the incident and emergent light, d the thickness of the absorbing layer, c the concentration and k the absorption constant.

[1] Holmes, W. C., Ind. and Eng. Chem. *16*, 35; 1924.
[2] Speas, W. E., Phys. Rev. *31*, *569;* 1928.

In Fig. 1 curve 3 shows two bands of equal prominence. As the concentration is increased the band on the short wave-length side becomes

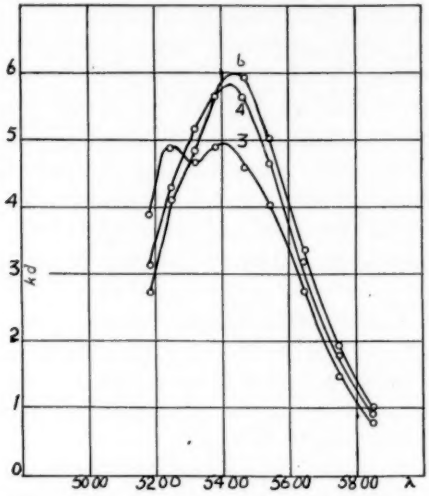

FIG. 1. *Liquid solutions. Absorption constant k times thickness d plotted against wave length. Concentrations (3) 2.13×10⁻⁷, (4) 3.50×10⁻⁷, (6) 5.48×10⁻⁷ gm mol/cc.*

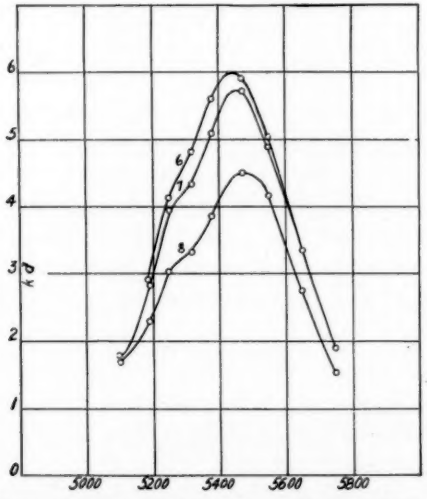

FIG. 2. *Liquid solutions. Absorption constant k times thickness d plotted against wave length. Concentrations (6) 5.48×10⁻⁷, (7) 8.14×10⁻⁷ (8) 12.1×10⁻⁷ gm mol/cc.*

less prominent while the long wave-length band has grown as curves 4 and 6 show. The crest of the long wave-length band is shifted towards longer wave lengths as the concentration is increased. A comparison of curves 7 and 8 with 6 in Fig. 2 shows a still further shift of the crests.

It will be noticed, however, that while k at first increases with the concentration it finally (curve 8, Fig. 2) shows a marked decrease.

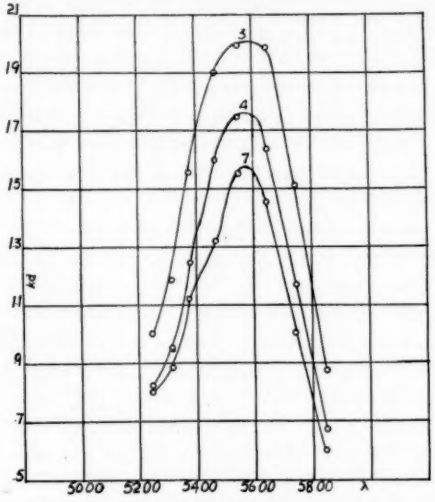

FIG. 3. *Solid solutions. Absorption constant k times thickness d plotted against wave length. Concentrations (3) 10.2×10^{-6} (4) 16.8×10^{-6} (7) 38.2×10^{-6} gm mol/cc.*

Fig. 3 represents the absorption curves for three solid solutions. As in the case of the liquid solutions the shift of the crests of the bands is towards longer wave lengths. The shift observed here for both liquid and solid solutions is in the same direction as found by Speas[2] for alcoholic solutions of rhodamin-B.

[2] Speas, W. E., Phys. Rev. *31*, 569; 1928.

Die optischen Instrumente (Brille, Lupe, Mikroskop, Fernrohr, Aufnahmelinse und ihnen verwandte Vorkehrungen), von *M. v. Rohr*. Vierte vermehrte und verbesserte Auflage. Seiten vi+130, mit 91 Abbildungen. Berlin: Verlag von Julius Springer, 1930.

Published originally in 1905 as "Bändchen 88" of Teubner's "Sammlung aus Natur und Geisteswelt," this deservedly popular manual on modern optical instruments has gone through four editions and comes to us now as an old friend in a new guise, not only revised and enlarged, but printed on a larger page, in Roman type, and more attractive in appearance in every way. The information which it contains is both accurate and concise, perhaps a little too meagre in some places; but the book is only intended to be a compendium on the subject and mathematical discussions are rigorously avoided as far as possible.

The fundamental conceptions of geometrical optics and the laws of optical imagery are briefly outlined in an introductory chapter; which is followed by a somewhat elementary chapter on the optical system of the eye, in which such subjects are also considered as accommodation, perspective representation of vision with both the passive and the mobile eye, binocular vision, and especially the use of spectacles for both far and near vision and for the correction of astigmatism. Like some other high authorities, Professor v. Rohr prefers not to consider spectacles as being optical instruments in the strict sense of the term but rather as a simple ocular device for enhancing the visual acuity of a poor eye whose vision for some reason or other is below normal. Thus a spectacle lens, even in a complicated form like telescopic spectacles or in case of a special construction like the so-called "Katral" lens for the aphakic eye, is nothing more than an aid to vision and not on the order of an optical instrument which is usually designed for persons with normal vision.

The real subject of the book is reserved, therefore, for the third and last chapter which comprises nearly three-fourths of the entire contents. Here the author takes up each type of instrument in turn, namely, the magnifying glass and the compound microscope, the telescope in its various forms and applications, and photographic lenses, lanterns and other forms of apparatus for projection. Incidentally, we find here descriptions also of many special appliances, *e.g.*, the projection-microscope (including its use in photography), the ultra-microscope, the prism telescope (including binocular field glasses), the stereoscope, the telestereoscope, the submarine periscope, the search-light (including automobile head-light), the range-finder, the ophthalmoscope, the magic lantern, the camera obscura, the telephotographic lens, apparatus for moving picture projection, the cystoscope and other optical contrivances used for diagnosis, surgical operations, etc.

On the other hand, whole classes of instruments are deliberately omitted as not coming within the province of this little treatise at all, especially such laboratory apparatus used mainly for investigations in physical optics as refractometers, spectroscopes, photometers and spectro-photometers, interferometers, polariscopes, colorimeters, fluoroscopes, x-ray equipments, etc., as well as a number of clinical instruments, for example, like the ophthalmometer, perimeter, stereo-campimeter, etc.

Modern optical instruments had their real beginning with the invention of the telescope and microscope early in the 17th century. An instrument of a totally different character from either of them was the magic lantern which was invented soon afterwards but which for a long time was little more than an ingenious and pleasing toy enabling a number of spectators to see a magnified image of a transparent slide projected on a screen, whereas the telescope or microscope was confined to the use of one single individual at a time. In the early days optical instruments came therefore to be divided naturally in two classes, namely, those which, having to be applied to the eye, were intended primarily for subjective use only, and those which like the magic lantern were more or less independent of any particular observer and designed to give an objective picture visible to all who happened to be within range of the apparatus. A new era began with the discovery of photography in the 19th century, especially with the invention of the daguerreotype process in France in 1839 and the development of the photographic negative in England in 1841; which gave an entirely new incentive to the design and construction of optical instruments for all kinds of reproductions and pictorial representations. From that time onward optical instruments began to flourish and develop as never before; and if a reader wishes to follow the remarkable achievements in the great province of applied optics he cannot have a better guide than this little volume which is filled with historical notes on the subject and accurate chronological data concerning the origin of each new improvement. Professor v. Rohr was himself a disciple of Abbe, and his whole life has been spent in association with the great optical firm of Carl Zeiss in Jena; he is therefore perfectly at home when it comes to relating the history of optics which is a subject on which he is unquestionably the greatest living authority.

So manifold and diversified are the uses of optics nowadays in all the arts of peace and war that it is practically impossible to frame a satisfactory definition that is broad enough to embrace every type of optical instrument. As we ordinarily think of such constructions, certainly one of their main functions consists in the portrayal of an object by means of a so-called optical image that will enable a single observer or possibly a whole company of spectators to obtain a conception of the prototype or original, such as under the circumstances could not have been obtained at all without the intervention of the instrument. Vague and incomprehensive as this definition

obviously is, it is open to a more serious objection, namely, as to what is meant precisely by an optical image; for this is a term that needs to be defined also, although it is not easy to do so. Thus, for example, the image that is focused on a screen by an achromatic lens is essentially different in formation from the optical projection on the sensitive plate of a pinhole camera, where the effect is produced by divergent bundles of rays of such narrow aperture that the consequent lack of sharpness is scarcely discernible in the reproduction. But even in the former case in which the bundles of image-rays are convergent, it is impossible for the screen to be in focus for all parts of the solid object at the same time, and consequently the representation on the screen will be at best no more than a perspective copy of the object. This is equally true in viewing an object through a magnifying glass, for what we actually see in this case is not so much the geometrical image hanging in space as a perspective projection of this image on an imaginary surface that is conjugate to the retina of the eye. These are some of the difficulties that are encountered at the outset when we try to define these terms with a view to classifying optical instruments according to some logical method.

In this volume, as also in previous publications, Professor v. Rohr offers a new classification which, it must be confessed, seems a little artificial in some ways, although it certainly has much to be said in its favour. Thus he divides optical instruments in two main groups, namely: 1. "Die verdeutlichenden Instrumente," which enable us to see an object more fully, above all more clearly and more distinctly, than we could do without their aid; and 2. "Die widerholenden Instrumente," which simply reproduce a copy or picture of the object that enables a spectator, even when he has never seen the object itself and perhaps never had an opportunity of seeing it, to form a more or less just and accurate conception of it. The simplest illustration of an optical device of this latter character, perhaps too simple to be dignified as an optical instrument at all, is an ordinary looking glass:

"Oh wad some power the giftie gie us
To see oursel's as ithers see us!"

A plane mirror enables a person to see his own countenance, which is a feat he could never accomplish without artificial aid; and with a pair of such mirrors he can even see the back of his head. In a concave mirror he can not only see his own face but he will be able to see it more distinctly than he can see that of another individual under ordinary conditions, because the concave mirror reproduces and magnifies at the same time. Now magnification is in fact the characteristic property of the first group of instruments mentioned above whose typical representatives are the telescope and microscope; and hence it might be inferred that a concave mirror belonged to this class by virtue of its magnifying power, though from the other point of view

it might be considered quite as well as belonging to the second group also. In order to avoid any ambiguity of this kind, Professor v. Rohr comes to the rescue by drawing an arbitrary line between the two groups corresponding to a two-fold magnifying power: if the magnifying power of the instrument is below this line, it is to be classed as being in the second group of copying or reproducing instruments. On the other hand, to take another simple illustration, a cylindrical or conical mirror which gives a distorted image that does not resemble the original object cannot be said to come under either of Professor v. Rohr's two main divisions; but this appears to be a rather trivial objection, as such optical contrivances are as yet of comparatively little importance and perhaps scarcely worth mentioning at all in this connection. According to Professor v. Rohr, a projection-microscope is in the same class with an ordinary microscope, although the latter is used "subjectively" and the former "objectively." Thus the new mode of classification cuts squarely athwart the older one, leading in some instances (at least so it seems to the reviewer) to rather forced associations between types of construction that at first glance would certainly seem to have little in common.

JAMES P. C. SOUTHALL

Applications of Interferometry. By D. Ewart Williams, M. Sc. E. P. Dutton and Company, Inc., New York, 104 pages, 43 diagrams.

Methods of producing interference effects are treated by the author in two broad classes; division of wavefront and division of amplitude. Typical examples of the former are given in a description of the essentials of the Rayleigh interferometer and its application as a refractometer, the Michelson stellar interferometer and its counterpart the Gerhardt microscope interferometer. Briefly considered are the diffraction grating and the echelon.

In the division of amplitude class are considered thin-film interference effects and as an application the testing of length gauges, also the Jamin interferometer. A complete chapter is devoted to the Michelson interferometer and his application in determining the fine structure of spectral lines from visibility curves; also his use of the instrument in measurement of length standards in terms of light wave standards. The Michelson and Morley ether drift and Michelson's earth rotation experiments are briefly described. Kennedy's half shadow method of detecting small displacements of fringe displacements is briefly considered.

Two chapters are devoted to simultaneous division of amplitude and wavefront and to multiple beams. Under the latter the Fabry Perot interferometer and its application are considered in some detail.

For the important types discussed, expressions for intensity distribution, resolving power, etc. are derived. To cover such a broad subject in a small monograph, obviously much detail must be omitted but it should serve as

a valuable reference for those interested in the technical application of interference methods. Some twenty-four general and one hundred text references given, enable one to locate easily more comprehensive accounts of selected phases of the subject.

R. D. WYCKOFF

The principles and practice of ocular physical therapy for optometrists. By J. I. Kurtz, B.Sc., A.O., F.A.A.O. 250 pages, 51 illustrations. Published by the *American Journal of Optometry*, 1930. $4.50.

This book has been written for the purpose of introducing the principles and practice of ocular physical therapy to optometrists. The first portion of the book deals with the physics of high frequency currents and the physiological and therapeutic effects of these currents. The second portion is concerned with clinical applications of electrotherapy, describing the various techniques and their applications to various ocular and systemic dysfunctions. Part three is devoted to light therapy and psycho-therapy. The concluding portion of the volume deals with mechano-therapy, describing several methods for the mechanical treatment of strabismus. In addition to the material on mechano-therapy contributed by R. W. Bennett, there are chapters on "Consideration of the energy transformation in the body from an electrical standpoint" by J. O. Perrine, an electrical engineer in the research department of the American Telegraph and Telephone Company, and "Galvanism for optometrists" by H. Riley Spitler.

CHARLES SHEARD

NOTES

Society of Motion Picture Engineers to hold Spring Meeting in Hollywood.—The Spring meeting of the Society of Motion Picture Engineers will be held in Hollywood May 25th to 28th, according to announcement by W. C. Kunzman following a recent meeting of the Board of Governor.

O. M. Glunt, Chairman of the Papers Committee has already begun work in securing representative speaker and papers for the meeting.

The last meeting to be held in Hollywood was in the Spring of 1928 and with so many new developments brought about by sound in the studios since then, the coming convention is expected to be of great interest and value to the members of the Society.